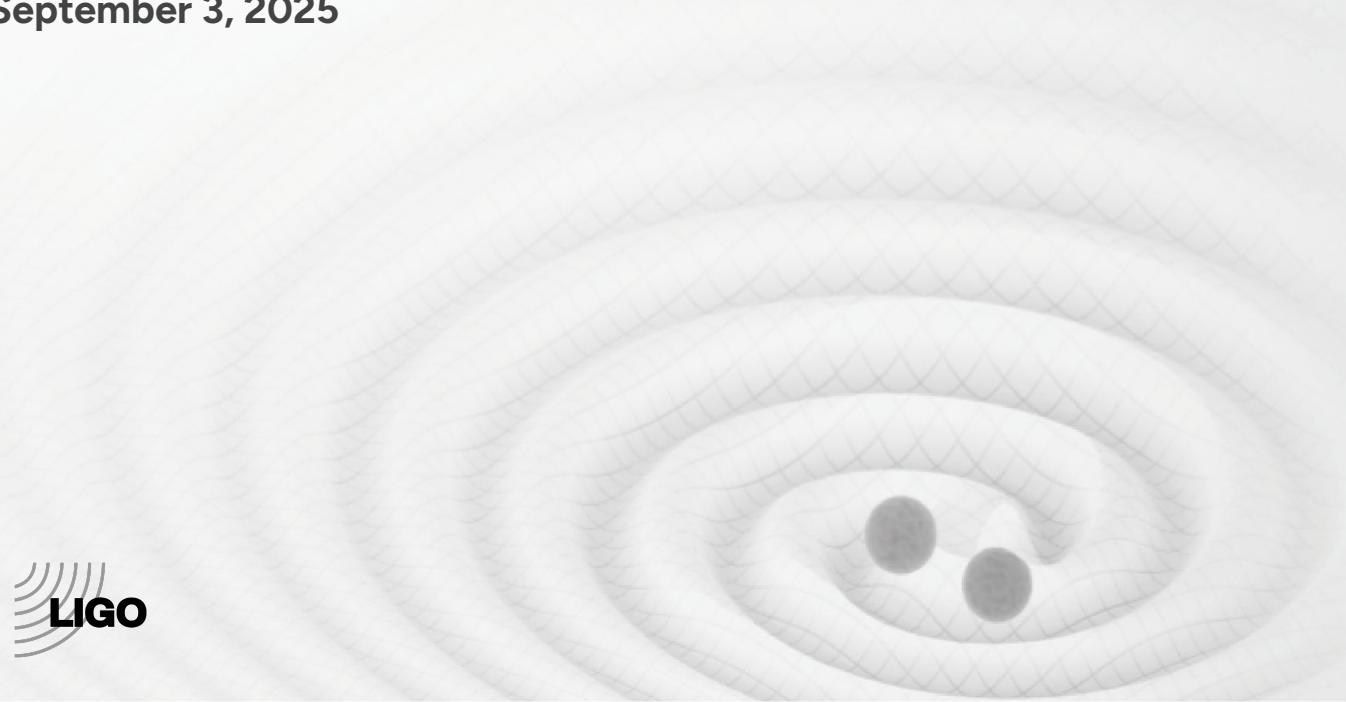


# A Differentiable Interferometer Simulator for the Computational Design of Gravitational Wave Detectors

Jonathan Klimesch, Yehonathan Drori, Rana X. Adhikari, Mario Krenn

Nikhef Institute on September 3, 2025



# Artificial Scientist Lab at the Tübingen Cybervalley



# Prof. Mario Krenn

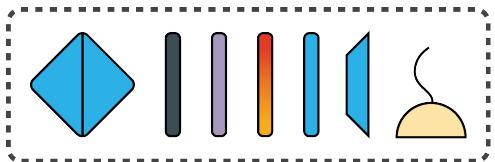
## The Team



A large, modern building with a distinctive white, faceted facade that resembles a crystalline or geometric pattern. The building has multiple levels and is set against a backdrop of a clear sky. In the foreground, several people are walking on a paved plaza in front of the building. A small tree stands near the entrance. To the right, another similar building is visible, and a bus is parked on the street. The overall atmosphere is clean and contemporary.

**Some Context ...**

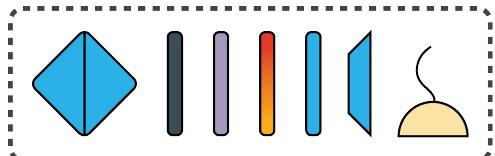
# Automated Search for New Quantum Experiments



Optical Element Toolbox



# Automated Search for New Quantum Experiments

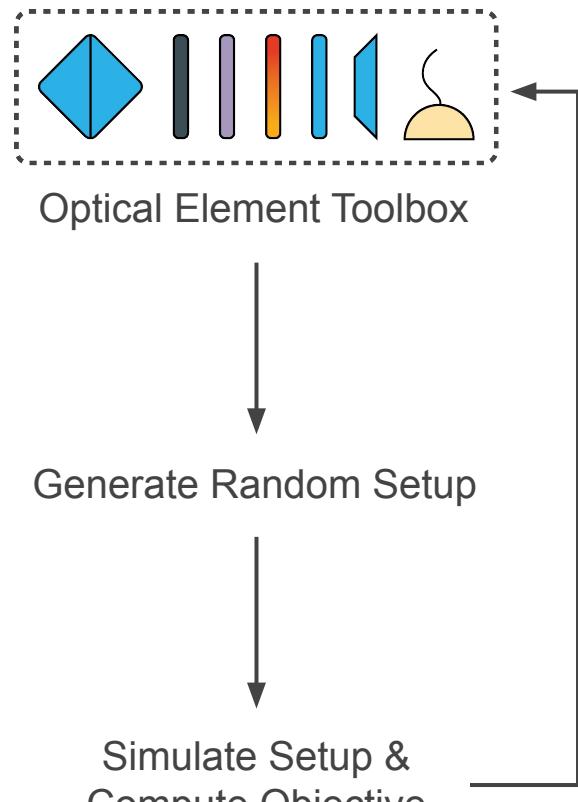


Optical Element Toolbox

$$\psi\rangle = \frac{1}{\sqrt{3}} (|0,0,0\rangle + |1,1,1\rangle + |2,2,2\rangle)$$

3D GHZ State

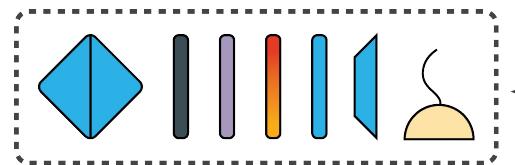
# Automated Search for New Quantum Experiments



$$\psi\rangle = \frac{1}{\sqrt{3}} (|0,0,0\rangle + |1,1,1\rangle + |2,2,2\rangle)$$

3D GHZ State

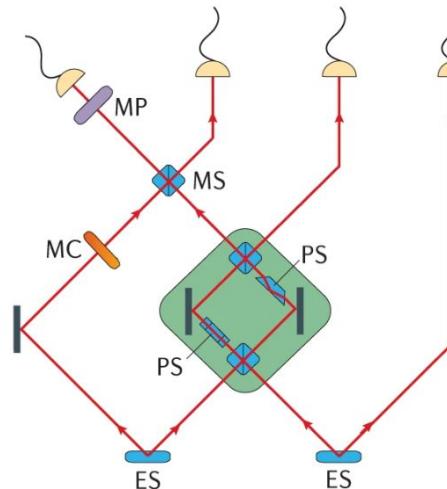
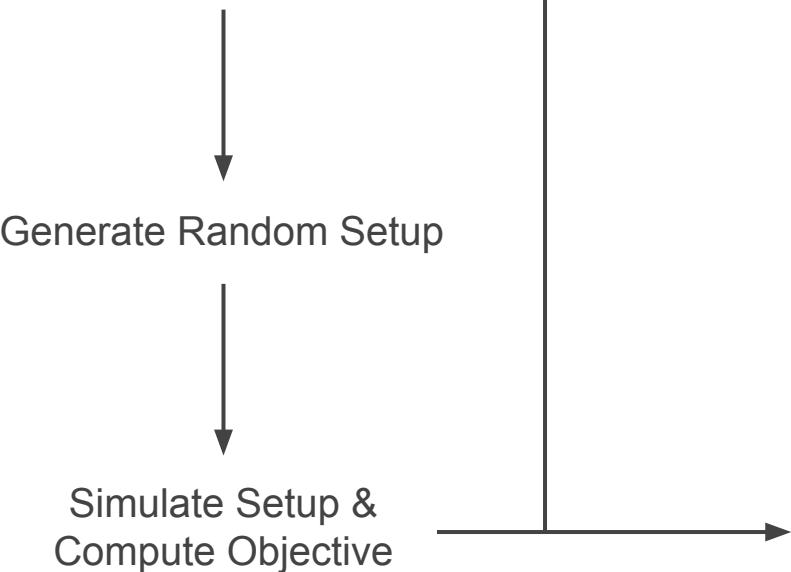
# Automated Search for New Quantum Experiments



Optical Element Toolbox

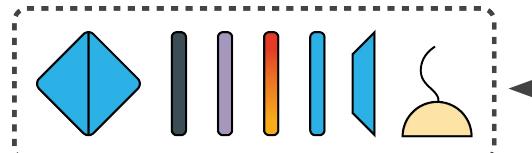
$$\psi\rangle = \frac{1}{\sqrt{3}} (|0,0,0\rangle + |1,1,1\rangle + |2,2,2\rangle)$$

3D GHZ State



[Krenn et al., Phys. Rev. Lett. 116 \(2016\)](#)

# Automated Search for New Quantum Experiments



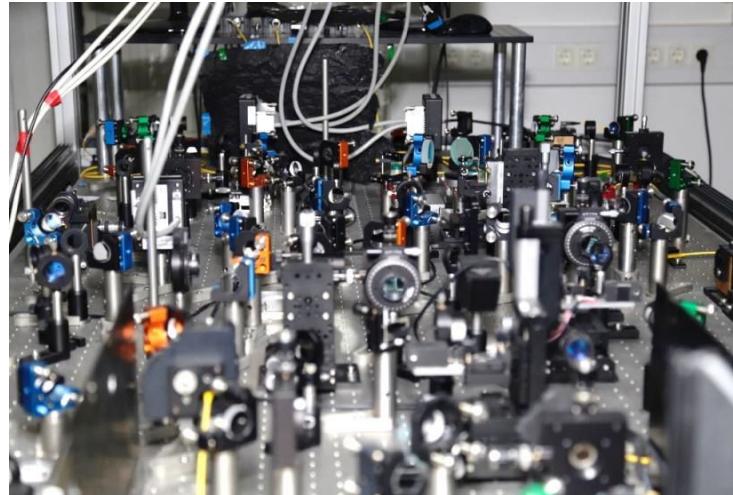
Optical Element Toolbox

$$\psi\rangle = \frac{1}{\sqrt{3}} (|0,0,0\rangle + |1,1,1\rangle + |2,2,2\rangle)$$

3D GHZ State

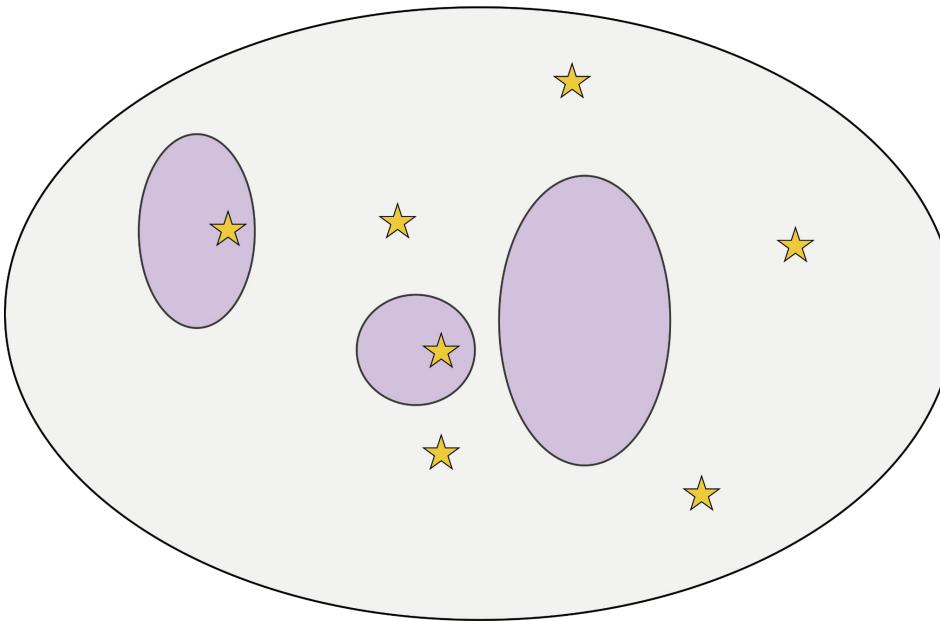
Generate Random Setup

Simulate Setup &  
Compute Objective



[Erhard et al., Nature Photonics 12, 759 \(2018\)](#)

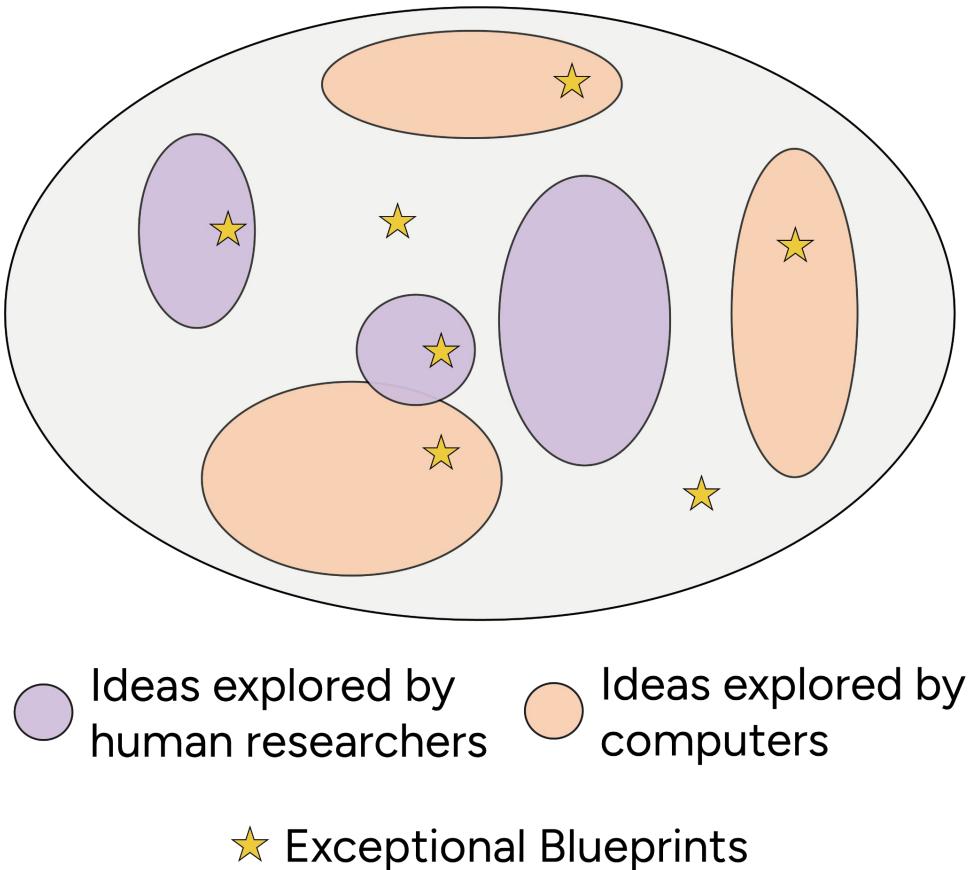
# Human Exploration



 Ideas explored by  
human researchers

 Exceptional Blueprints

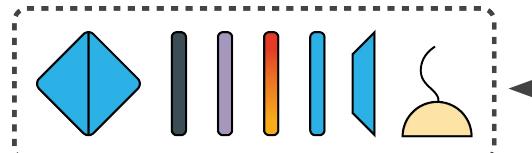
# Computational Exploration





**Can AI find unorthodox, but useful  
physics experiment designs?**

# Automated Search for New Quantum Experiments



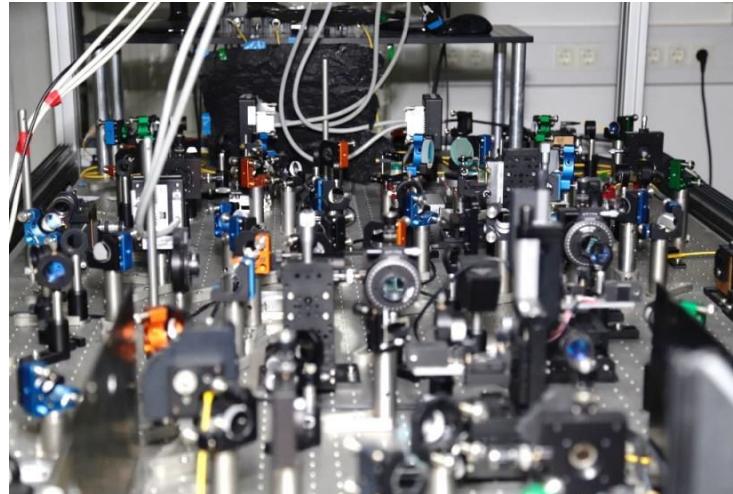
Optical Element Toolbox

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3D GHZ State

Generate Random Setup

Simulate Setup &  
Compute Objective



[Erhard et al., Nature Photonics 12, 759 \(2018\)](#)

# Automated Search for New Quantum Experiments



Optical Element Toolbox

JULY 2, 2021 | 8 MIN READ

## AI Designs Quantum Physics Experiments beyond What Any Human Has Conceived

Originally built to speed up calculations, a machine-learning system is now making shocking progress at the frontiers of experimental quantum physics

BY ANIL ANANTHASWAMY EDITED BY LEE BILLINGS

Generate Random Setup

Simulate Setup & Compute Objective



[Erhard et al., Nature Photonics 12, 759 \(2018\)](#)

# Automated Search for New Quantum Experiments



Simulate Setup &  
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[Erhard et al., Nature Photonics 12, 759 \(2018\)](#)

# Automated Search for New Quantum Experiments



Simulate Setup &  
Compute Objective



OPEN ACCESS

## Digital Discovery of Interferometric Gravitational Wave Detectors

Mario Krenn<sup>1,\*†</sup>, Yehonathan Dror<sup>2,\*‡</sup>, and Rana X Adhikari<sup>1,§</sup>

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Phys. Rev. X 15, 021012 – Published 11 April, 2025

DOI: <https://doi.org/10.1103/PhysRevX.15.021012>

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Citations 3

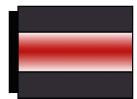
Show metrics ▾

### Abstract

Gravitational waves, detected a century after they were first theorized, are space-time distortions caused by some of the most cataclysmic events in the Universe, including black hole mergers and supernovae. The successful detection of these waves has been made possible by ingenious detectors designed by human experts. Beyond these successful designs, the vast space of experimental configurations remains largely unexplored, offering an exciting territory potentially rich in innovative and unconventional detection strategies. Here, we demonstrate an intelligent computational strategy to explore this enormous space, discovering unorthodox topologies for gravitational wave detectors that significantly outperform the currently best-known designs under realistic experimental constraints. This increases the potentially observable volume of the Universe by up to 50-fold. Moreover, by analyzing the best solutions from our superhuman algorithm, we uncover entirely new physics ideas at their core. At a bigger picture, our methodology can readily be extended to AI-driven design of experiments across wide domains of fundamental physics, opening fascinating new windows into the Universe.

[Erhard et al., Nature Photonics 12, 759 \(2018\)](#)

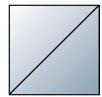
# Discrete-Continuous Search Space



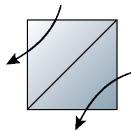
Laser (Power)



Mirror (Reflectivity, Tuning)



Beamsplitter (Reflectivity)



Directional Beamsplitter



Detector



Squeezer (dB, Angle)



Space (Length)

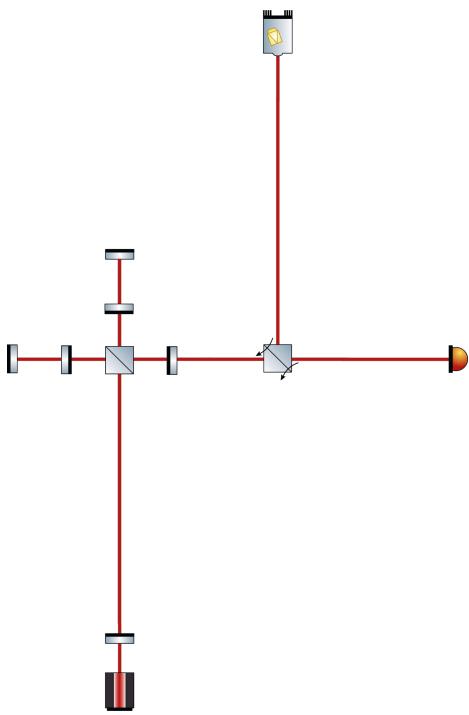


Suspension (Mass)

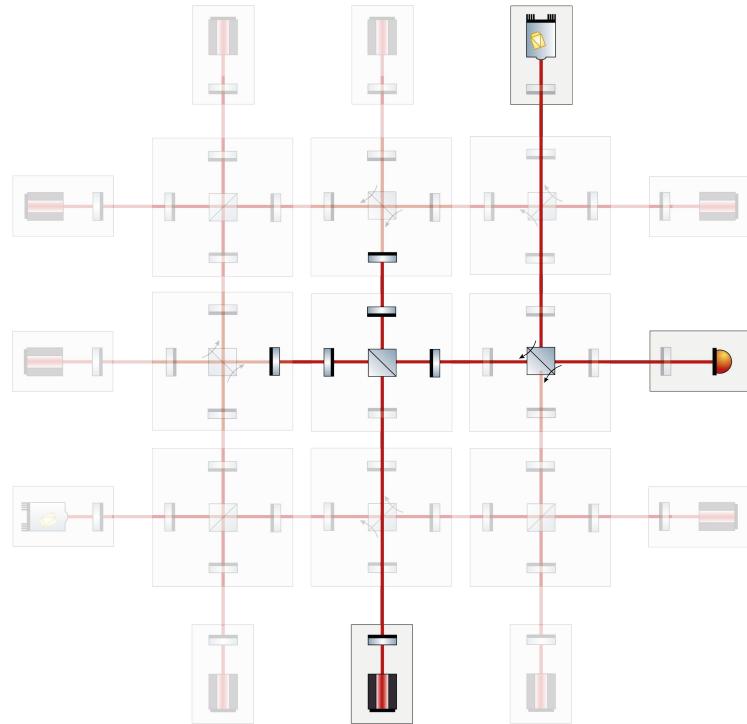
Where should we place which component?

Which parameters should we choose?

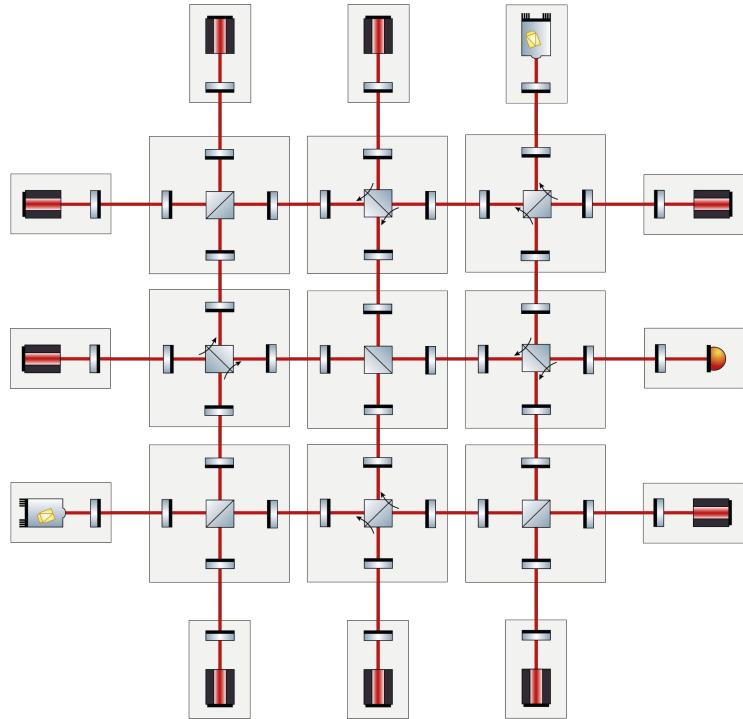
# Simplified aLIGO Setup



# Continuous and Highly Expressive Search Space



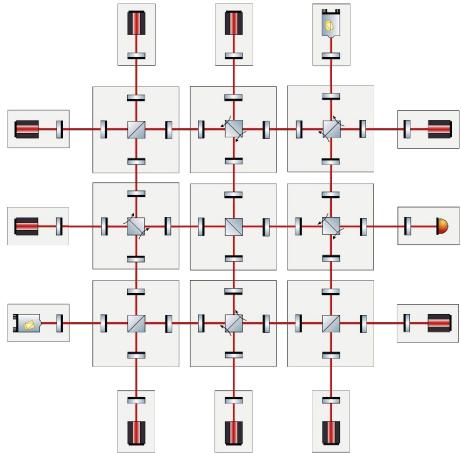
# Continuous and Highly Expressive Search Space



Quasi-Universal Interferometer  
(UIFO)

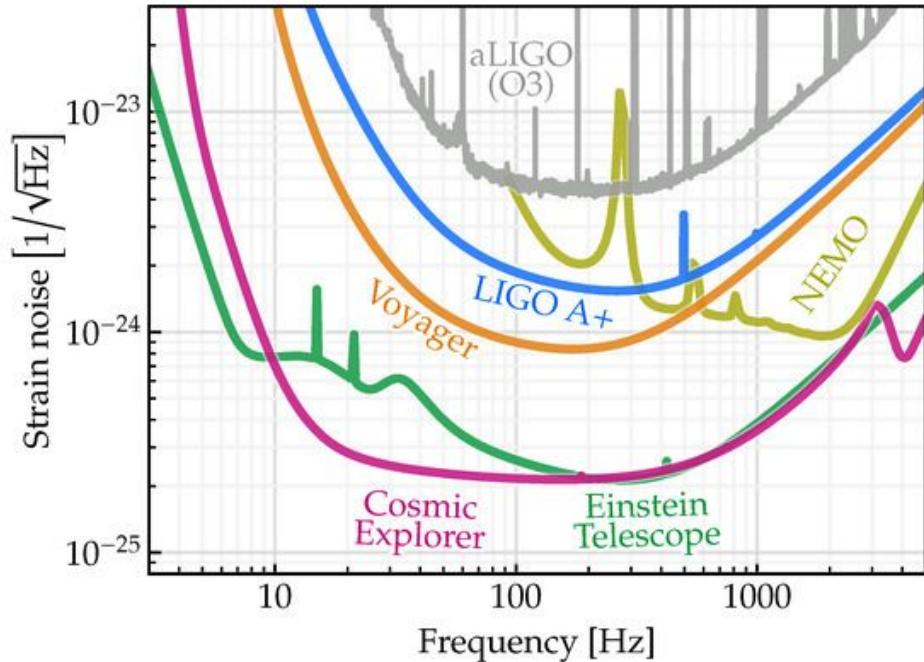
# Ingredients

## 1. Continuous Search Space



What is our optimization goal?

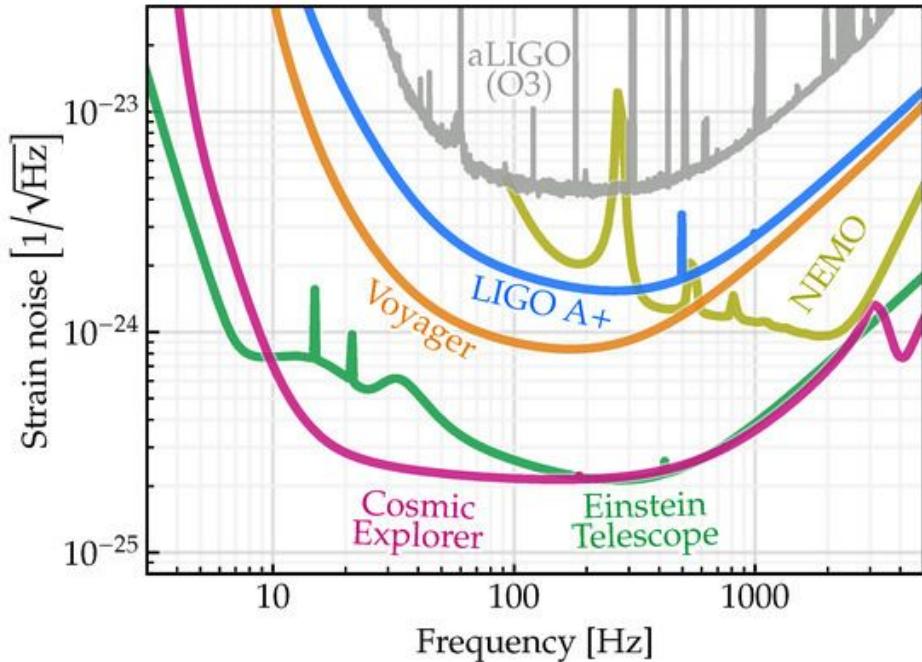
# The Objective Function



$$\mathcal{L}_{\text{Strain}} = \int_{f_0}^{f_1} \log(S(f)) df$$

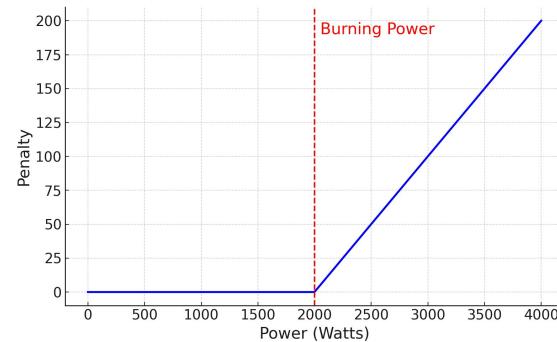
[Hall E. et al., Galaxies 2022, 10\(4\), 90 \(2022\)](#)

# The Objective Function



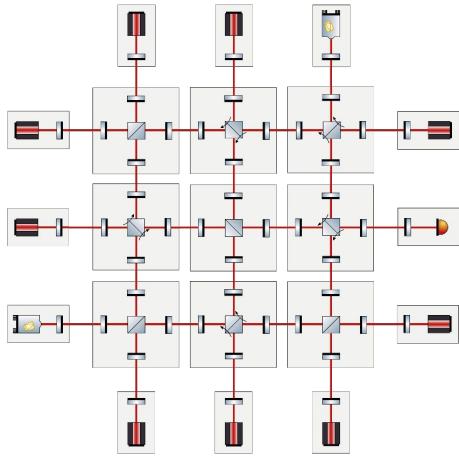
[Hall E. et al., Galaxies 2022, 10\(4\), 90 \(2022\)](#)

$$\mathcal{L}_{\text{Strain}} = \int_{f_0}^{f_1} \log(S(f)) df$$
$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{Strain}} + \alpha \cdot \text{Penalties}$$



# Ingredients

1. Continuous Search Space



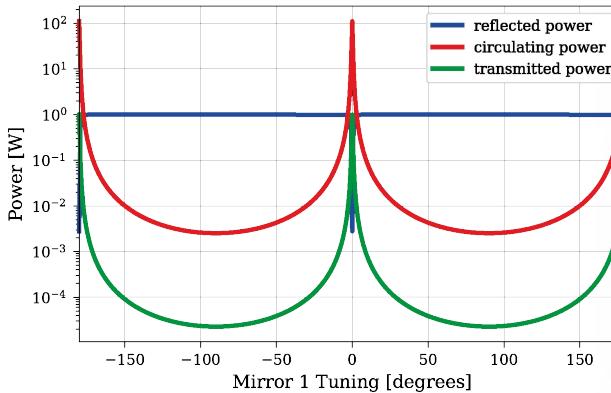
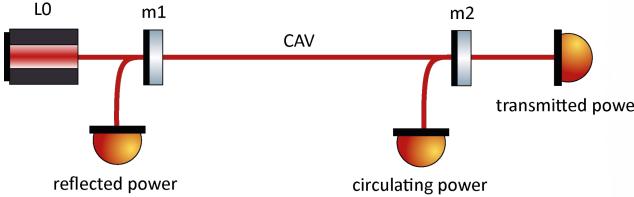
2. Objective Function

$$\mathcal{L}_{\text{Strain}} = \int_{f_0}^{f_1} \log(S(f)) df$$

**How do we evaluate the objective function?**

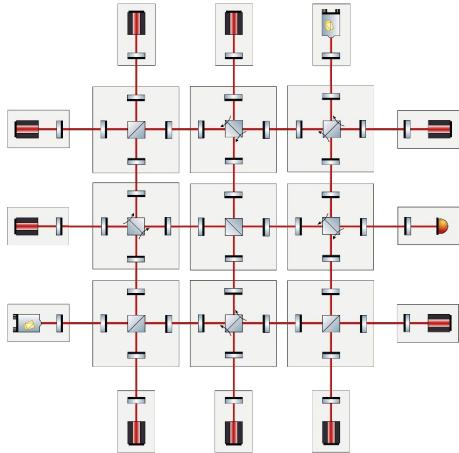
# Finesse - Frequency domain interferometer simulation software

```
1 import finesse
2 finesse.configure(plotting=True)
3
4 kat = finesse.Model()
5 kat.parse(
6     """
7 # Add a Laser named L0 with a power of 1 W.
8 l L0 P=1
9
10 # Space attaching L0 <-> m1 with length of 0 m (default).
11 s s0 L0.p1 m1.p1
12
13 # Highly reflective input mirror of cavity
14 m m1 R=0.99 T=0.01
15
16 # Intra-cavity space with length of 1 m.
17 s CAV m1.p2 m2.p1 L=1
18
19 # Highly reflective end mirror of cavity.
20 m m2 R=0.991 T=0.009
21
22 # Power detectors on reflection, circulation and transmission.
23 pd reflected_power m1.p1.o
24 pd circulating_power m2.p1.i
25 pd transmitted_power m2.p2.o
26 """
27 )
28
29 out = kat.run("xaxis(m1.phi, lin, -180, 180, 400)")
30 out.plot(logy=True)
```



# Ingredients

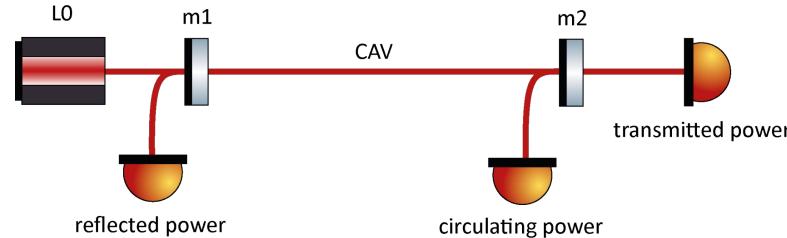
## 1. Continuous Search Space



## 2. Objective Function

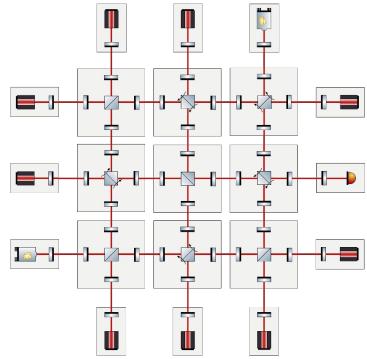
$$\mathcal{L}_{\text{Strain}} = \int_{f_0}^{f_1} \log(S(f)) df$$

## 3. Simulator



How do we optimize this?

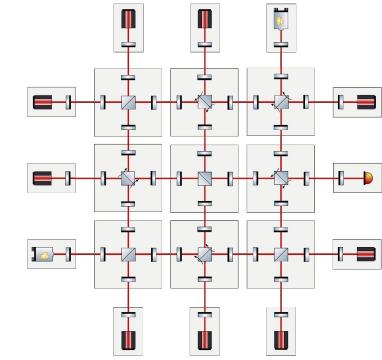
# Automated Search for New Gravitational Wave Detectors



Generate Random  
UIFO

$$\mathcal{L}_{\text{Strain}} = \int_{f_0}^{f_1} \log(S(f)) df$$

# Automated Search for New Gravitational Wave Detectors



Generate Random  
UIFO

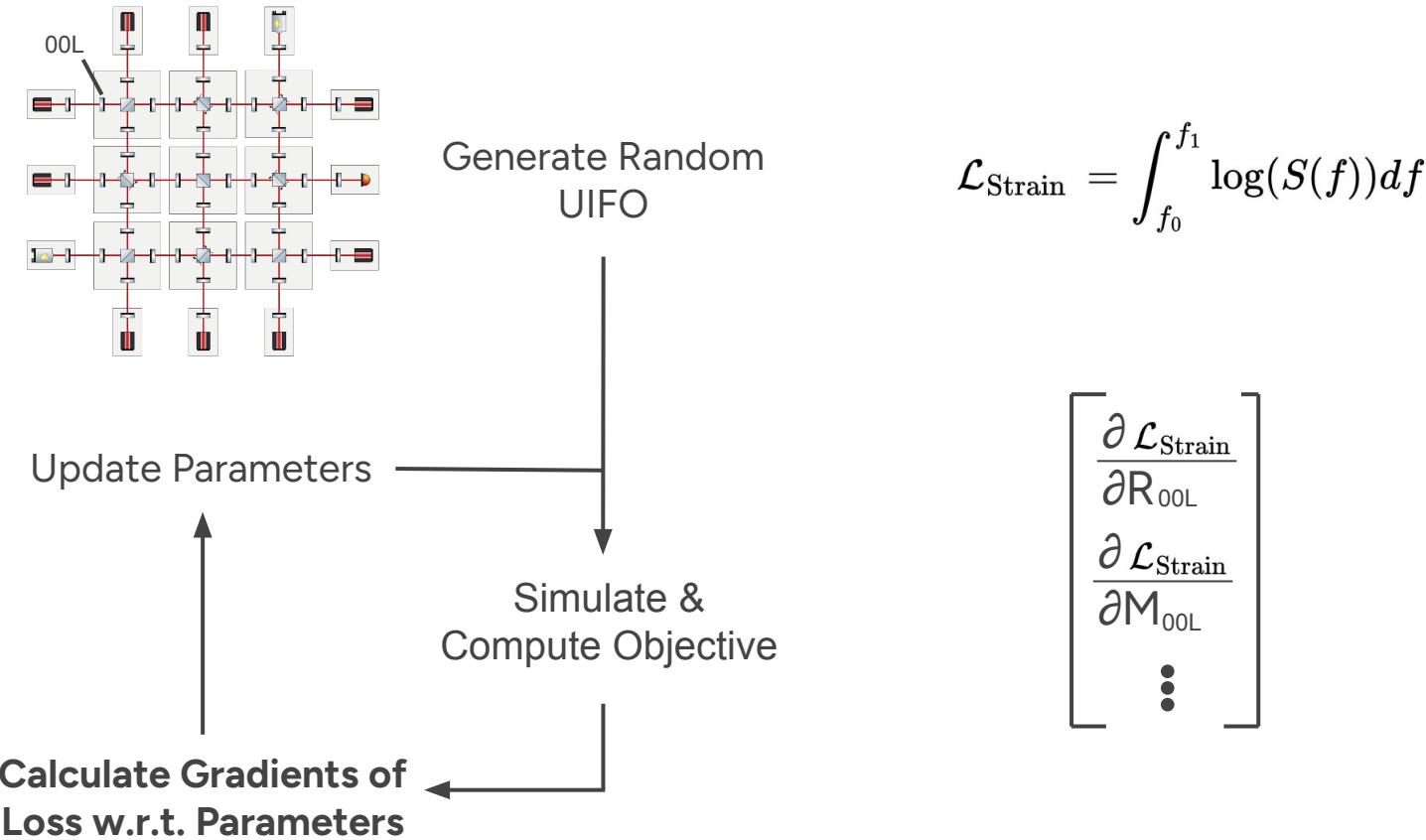
$$\mathcal{L}_{\text{Strain}} = \int_{f_0}^{f_1} \log(S(f)) df$$

Update Parameters

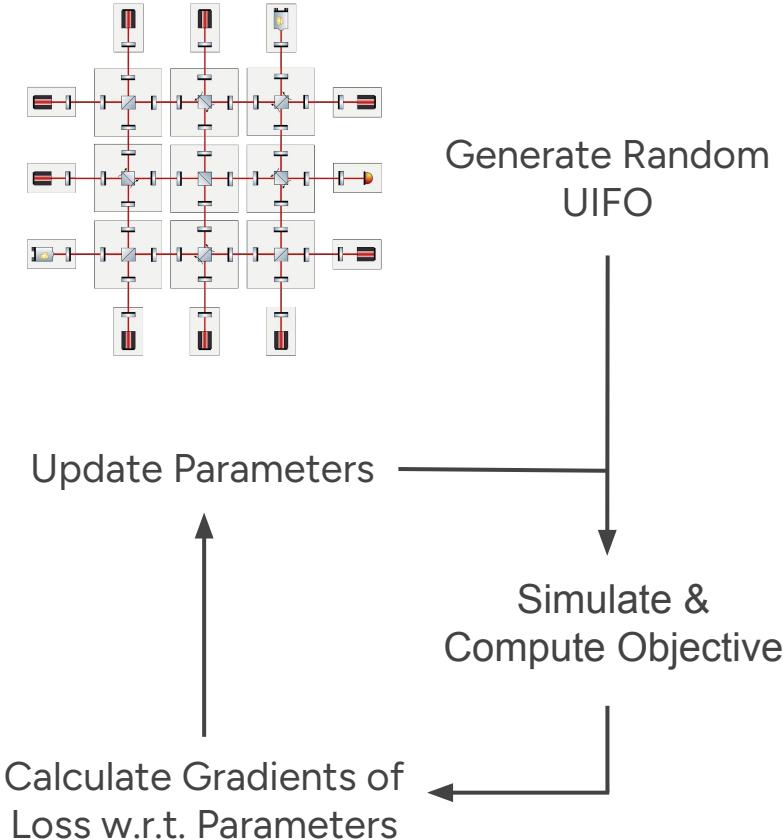
Simulate &  
Compute Objective

Calculate Gradients of  
Loss w.r.t. Parameters

# Automated Search for New Gravitational Wave Detectors

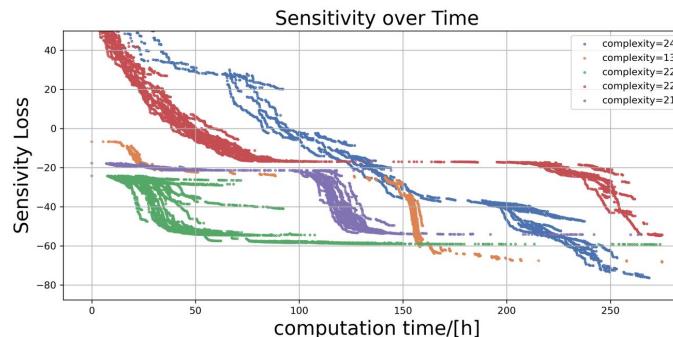


# Automated Search for New Gravitational Wave Detectors



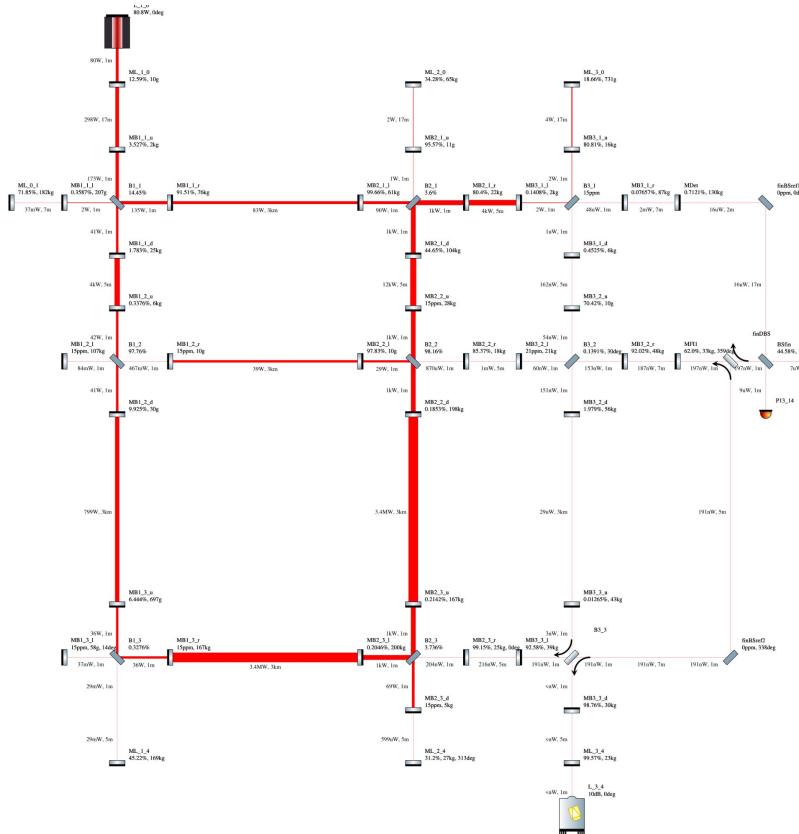
$$\mathcal{L}_{\text{Strain}} = \int_{f_0}^{f_1} \log(S(f)) df$$

BFGS optimizations



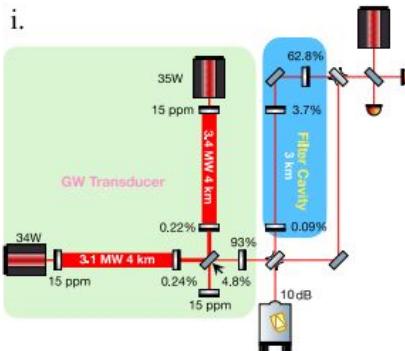
[Krenn et al., Phys. Rev. X 15 \(2025\)](#)

# Discovered Topologies - Raw Solutions

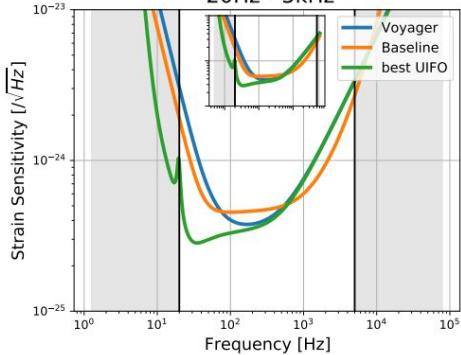


# Conceptualized UIFOs

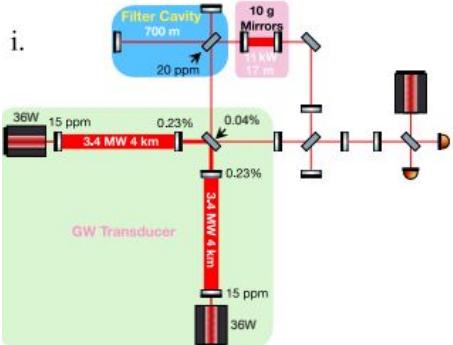
A) Broadband (30 Hz - 5 KHz)



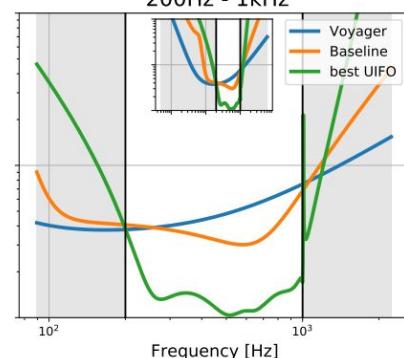
**Broadband**  
20Hz - 5kHz



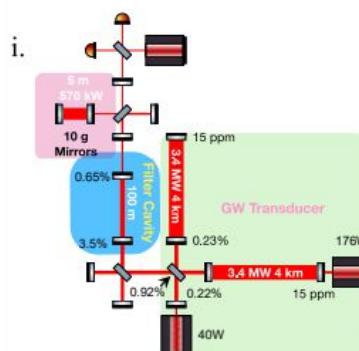
B) Supernova (200 Hz - 1 KHz)



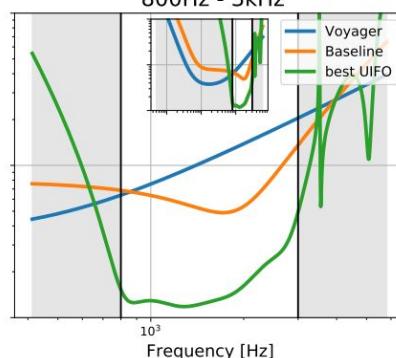
**Supernova**  
200Hz - 1kHz



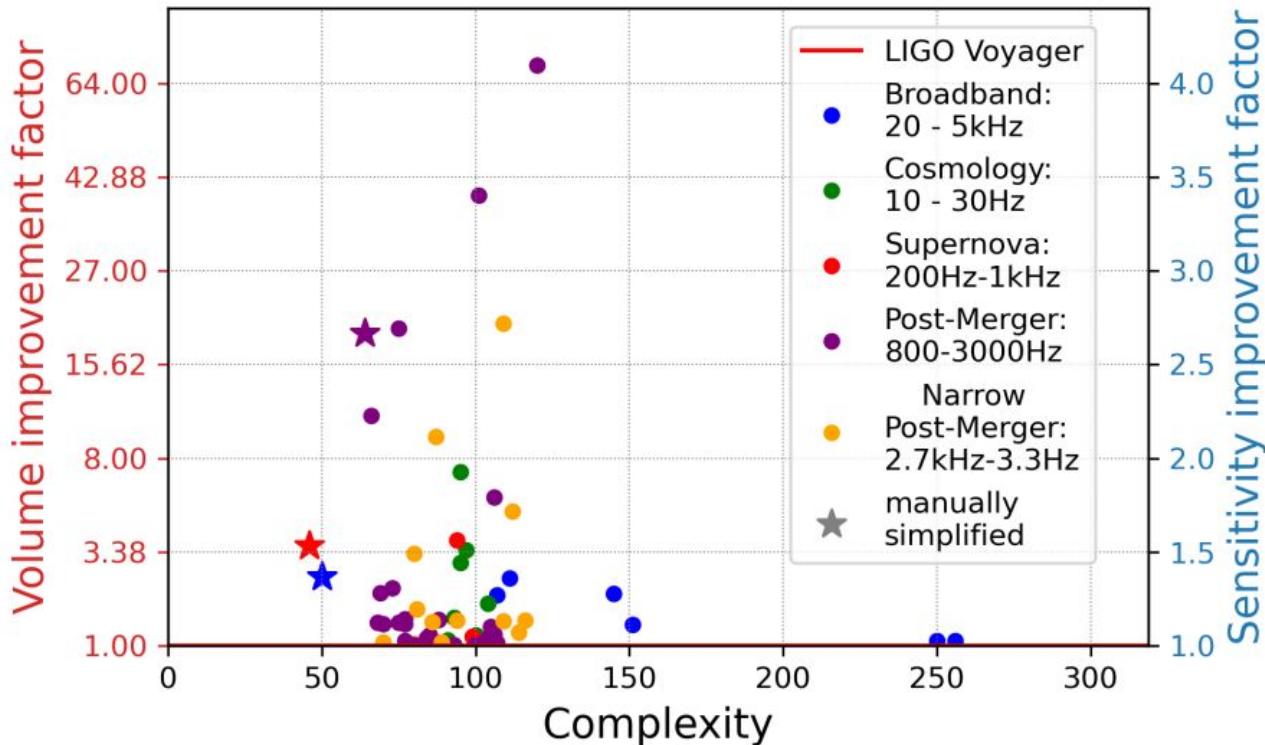
C) Postmerger (800 Hz - 3 KHz)



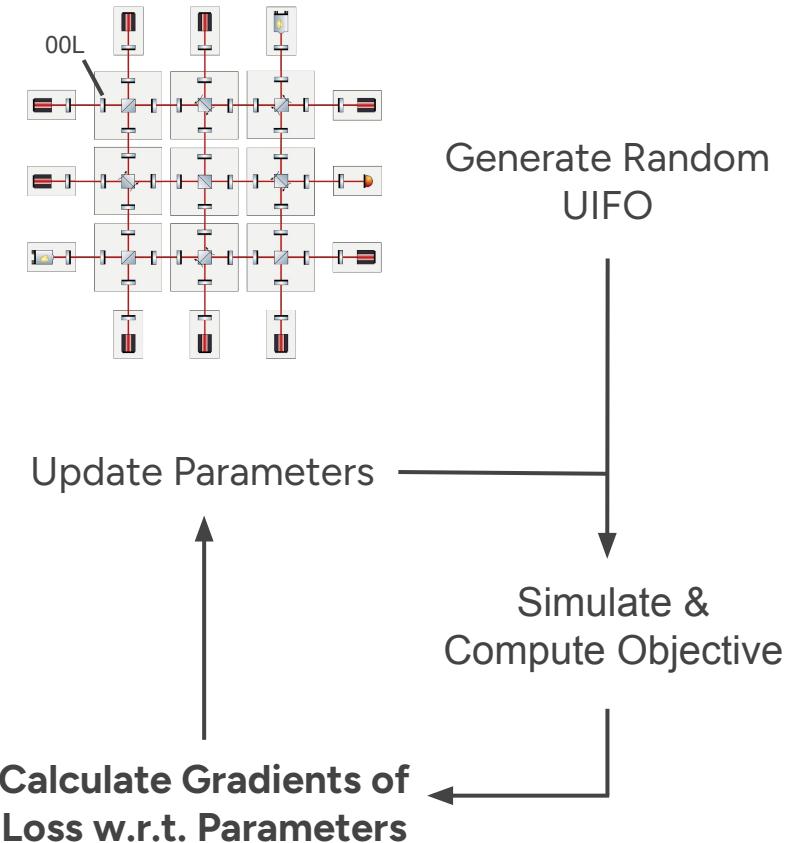
**Post – Merger**  
800Hz - 3kHz



# Gravitational Wave Detector Zoo



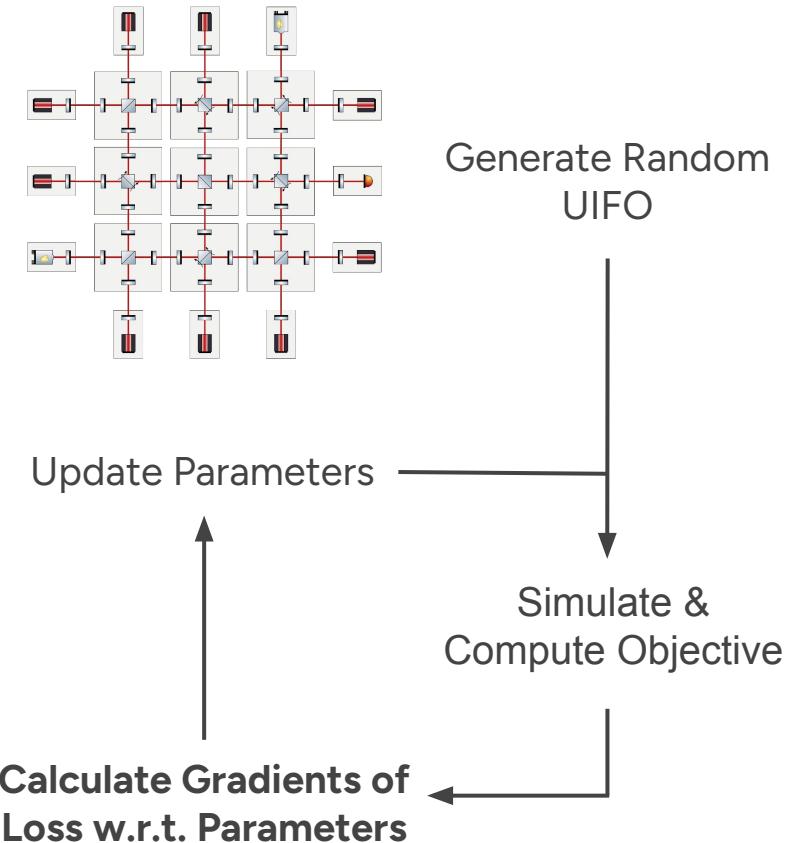
# Automated Search for New Gravitational Wave Detectors



$$\mathcal{L}_{\text{Strain}} = \int_{f_0}^{f_1} \log(S(f)) df$$

$$\begin{bmatrix} \frac{\partial \mathcal{L}_{\text{Strain}}}{\partial R_{00L}} \\ \frac{\partial \mathcal{L}_{\text{Strain}}}{\partial M_{00L}} \\ \vdots \end{bmatrix}$$

# Automated Search for New Gravitational Wave Detectors

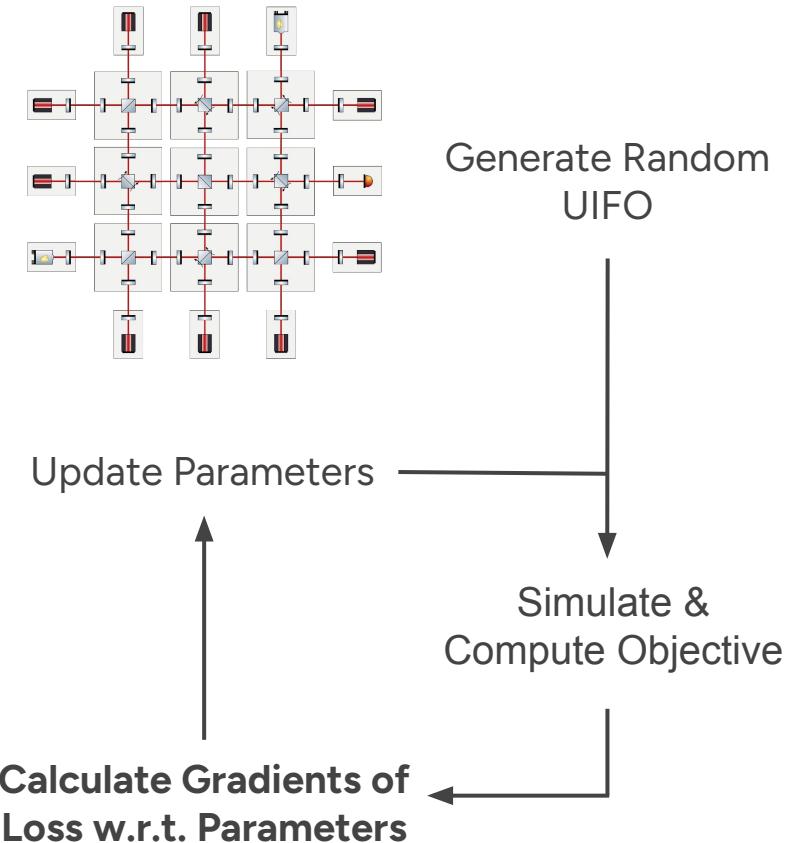


$$\mathcal{L}_{\text{Strain}} = \int_{f_0}^{f_1} \log(S(f)) df$$

$$f(x, y, z) = x + xy + z^2y$$

$$\frac{\partial f}{\partial x} \approx \frac{f(x+h, y, z) - f(x, y, z)}{h}$$

# Automated Search for New Gravitational Wave Detectors



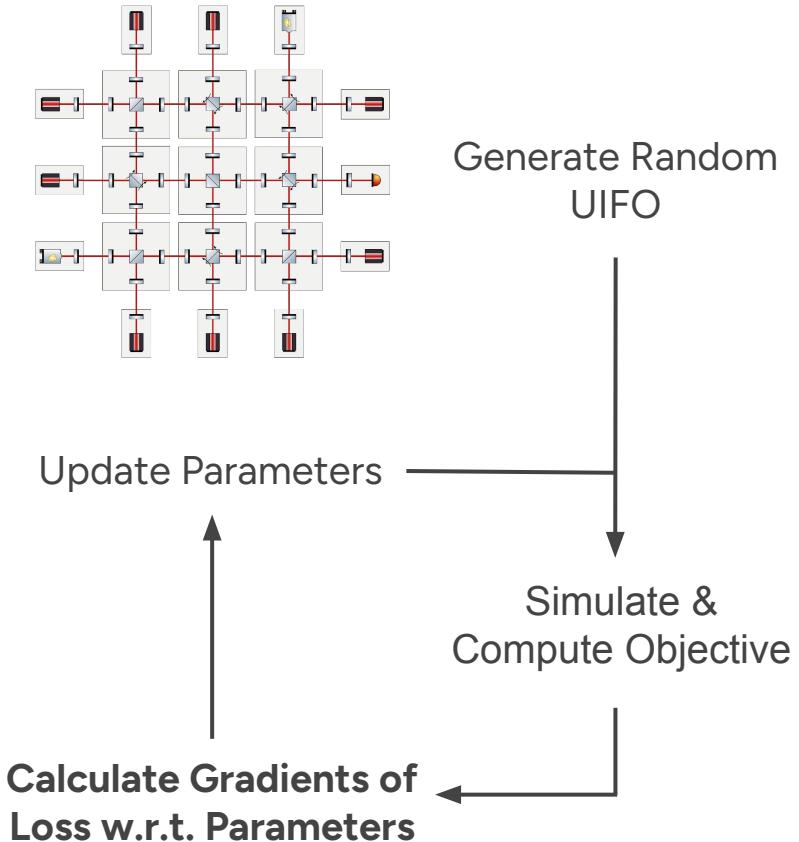
$$\mathcal{L}_{\text{Strain}} = \int_{f_0}^{f_1} \log(S(f)) df$$

$$f(x, y, z) = x + xy + z^2y$$

$$\frac{\partial f}{\partial x} \approx \frac{f(x+h, y, z) - f(x, y, z)}{h}$$

$$\frac{\partial f}{\partial y} \approx \frac{f(x, y+h, z) - f(x, y, z)}{h}$$

# Automated Search for New Gravitational Wave Detectors



$$\mathcal{L}_{\text{Strain}} = \int_{f_0}^{f_1} \log(S(f)) df$$

$$f(x, y, z) = x + xy + z^2y$$

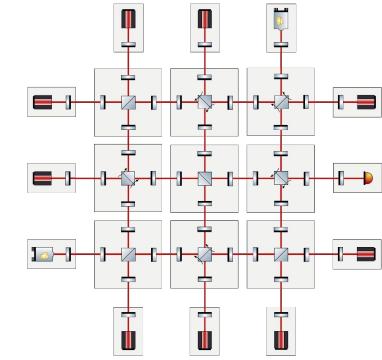
$$\frac{\partial f}{\partial x} \approx \frac{f(x+h, y, z) - f(x, y, z)}{h}$$

$$\frac{\partial f}{\partial y} \approx \frac{f(x, y+h, z) - f(x, y, z)}{h}$$

$$\frac{\partial f}{\partial z} \approx \frac{f(x, y, z+h) - f(x, y, z)}{h}$$

$$\nabla f \approx \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

# Automated Search for New Gravitational Wave Detectors



Generate Random  
UIFO

Computational cost:  
**1.5 million CPU-hours**

Update Parameters

Simulate &  
Compute Objective

**Calculate Gradients of  
Loss w.r.t. Parameters**

$$f(x, y, z) = x + xy + z^2y$$

$$\frac{\partial f}{\partial x} \approx \frac{f(x+h, y, z) - f(x, y, z)}{h}$$

$$\frac{\partial f}{\partial y} \approx \frac{f(x, y+h, z) - f(x, y, z)}{h}$$

$$\frac{\partial f}{\partial z} \approx \frac{f(x, y, z+h) - f(x, y, z)}{h}$$

$$\nabla f \approx \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$



# **Can we make Finesse differentiable?**

## Making something differentiable

$$f(x, y, z) = x + xy + z^2y$$



# Making something differentiable

$$f(x, y, z) = x + xy + z^2y$$

Replace with  
learned model

$$\approx \hat{f}(x, y, z; \theta)$$

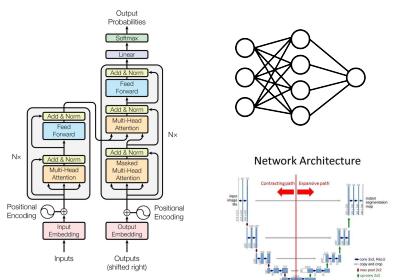


Figure 1: The Transformer - model architecture.

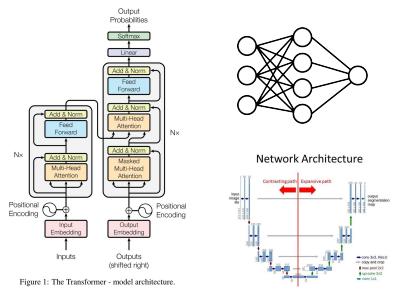
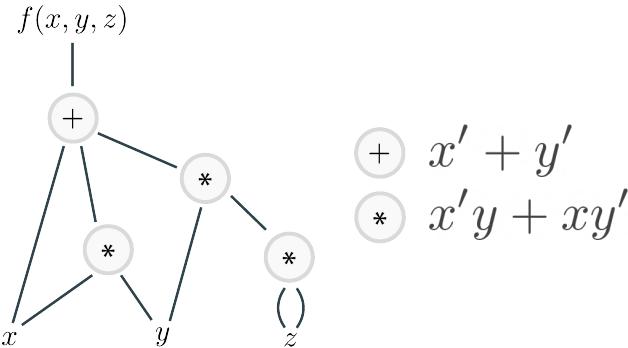
# Making something differentiable

$$f(x, y, z) = x + xy + z^2y$$

Replace with learned model

Rewrite in autodiff framework

$$\approx \hat{f}(x, y, z; \theta)$$



# Making something differentiable

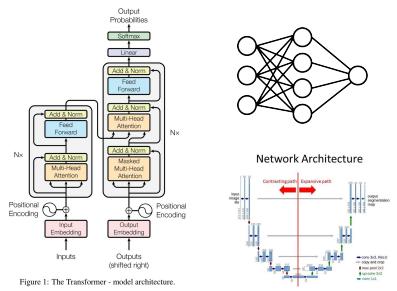
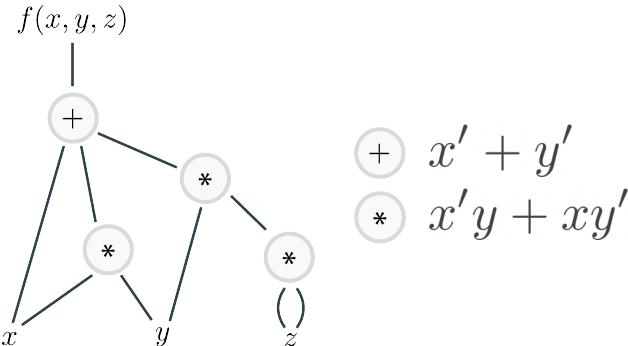
$$f(x, y, z) = x + xy + z^2y$$

Replace with learned model

Rewrite in autodiff framework

Augment source code for autodiff

$$\approx \hat{f}(x, y, z; \theta)$$

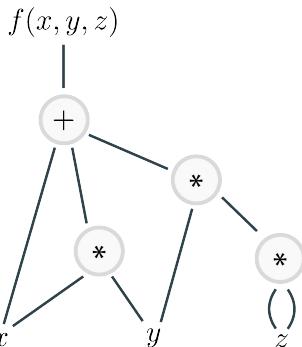


$\partial_{\partial_x}$  Enzyme AD Cla $\partial$   
Derivgrind TAPENADE

# Making something differentiable

$$f(x, y, z) = x + xy + z^2y$$

Rewrite in autodiff  
framework





# - High performance array computing

## Features:

- GPU support & vectorization
- Just-in-Time compilation
- Automatic Differentiation



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## Constraints:

- Only pure functions
- No conditionals
- Only static shapes



# - High performance array computing

## Features:

- GPU support & vectorization
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- Only static shapes

```
import jax

class Counter:
    def __init__(self):
        self.n = 0

    def count(self) -> int:
        self.n += 1
        return self.n

counter = Counter()
fast_count = jax.jit(counter.count)

# This results in 1, 1, 1!
for _ in range(3):
    print(fast_count())
```



# - High performance array computing

## Features:

- GPU support & vectorization
- Just-in-Time compilation
- Automatic Differentiation

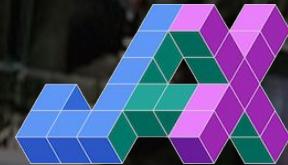
## Constraints:

- Only pure functions
- No conditionals
- Only static shapes

```
@jit
def f(x):
    if x < 3:
        return 3. * x ** 2
    else:
        return -4 * x
```

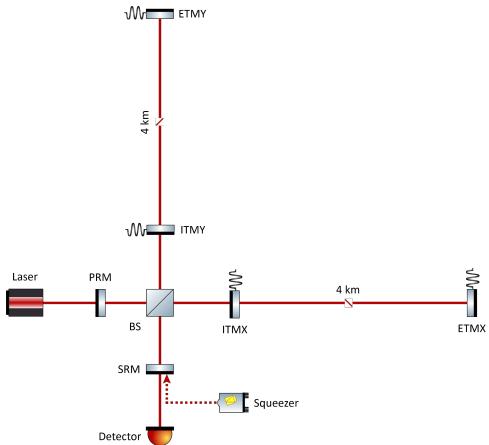
```
# This will fail!
f(2)
```

# Rewriting Finesse from Scratch in

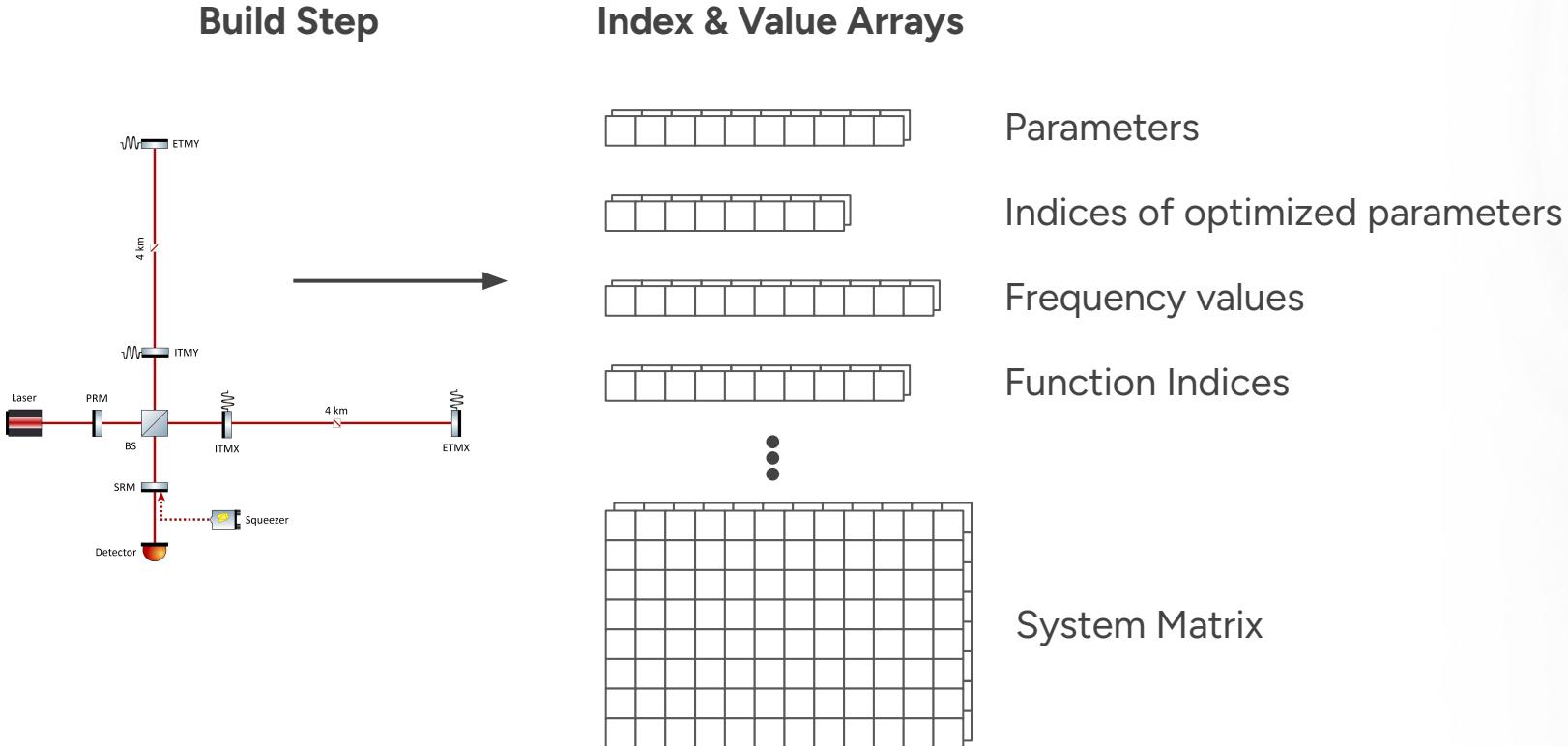


# Rewriting Finesse from Scratch in

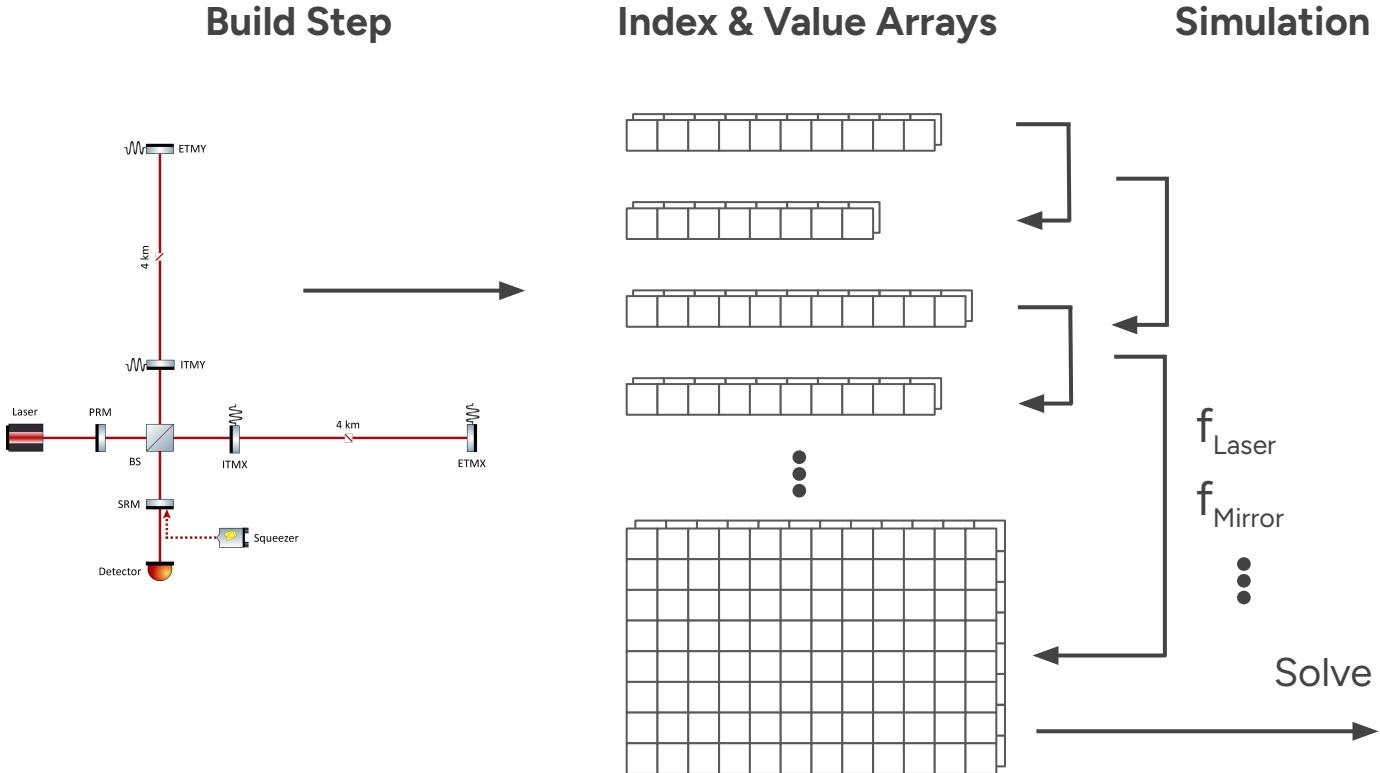
## Build Step



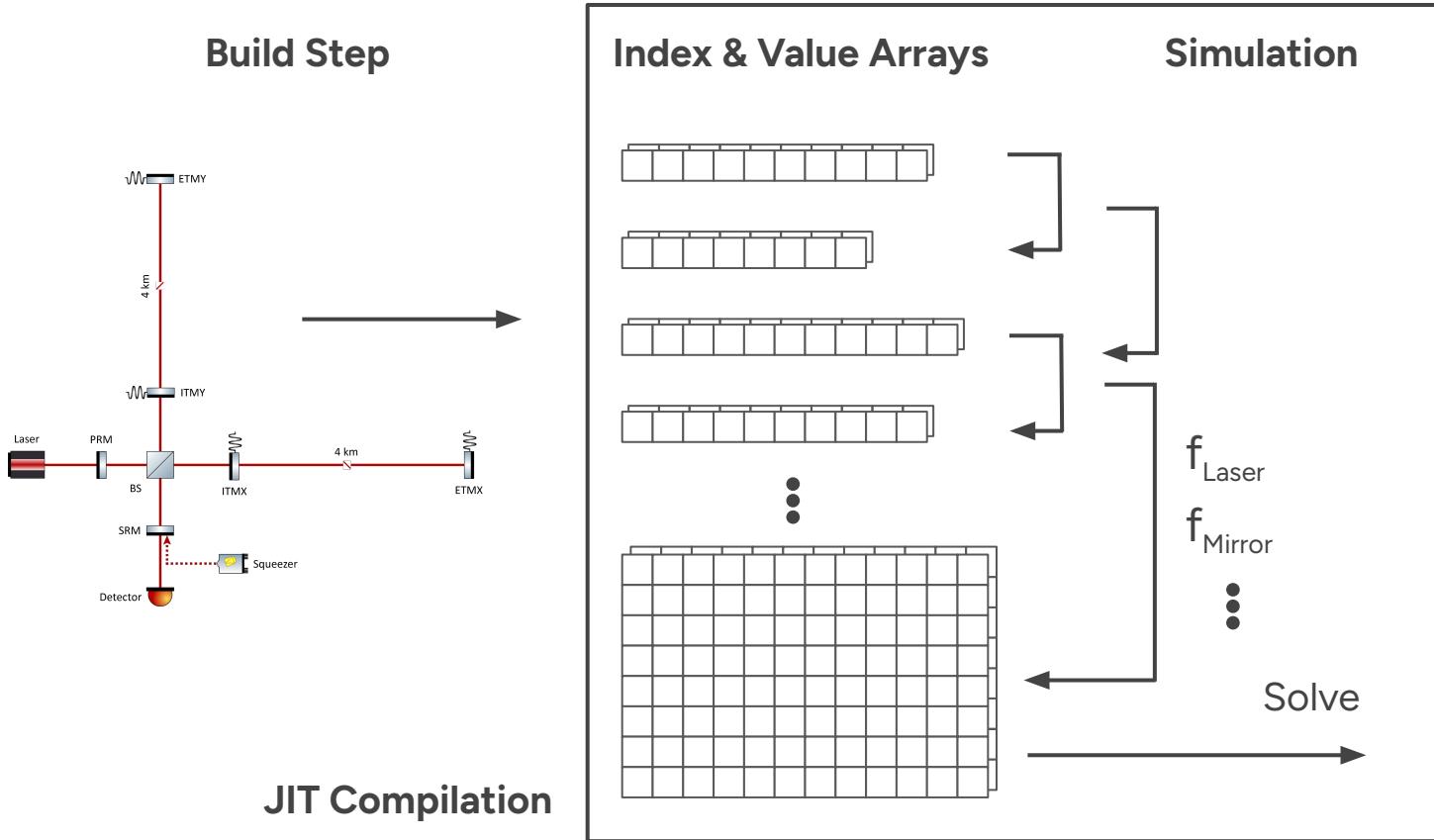
# Rewriting Finesse from Scratch in



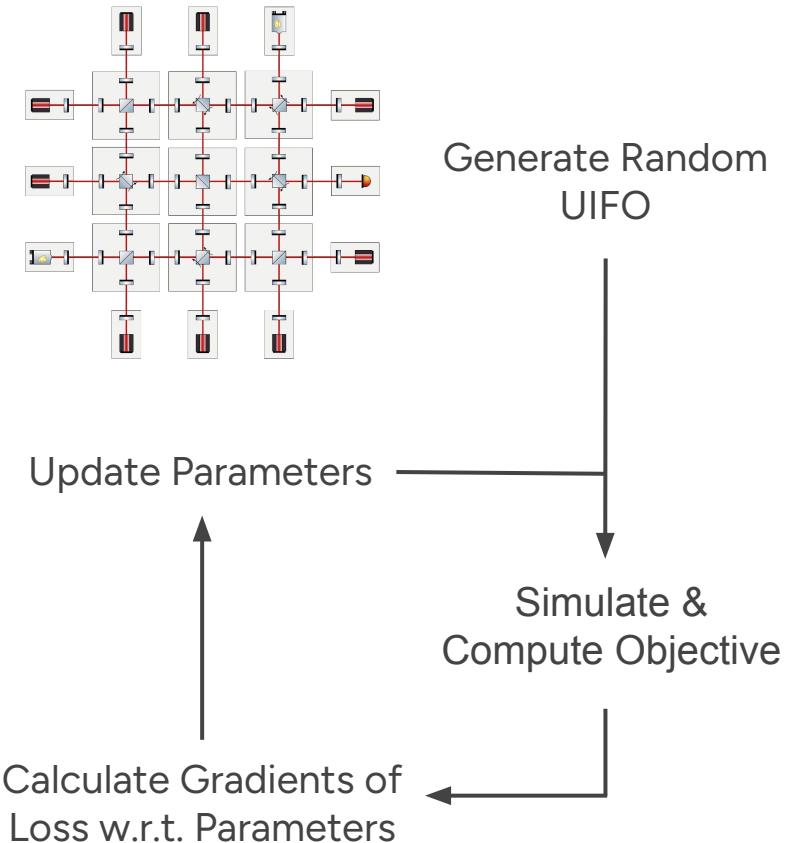
# Rewriting Finesse from Scratch in



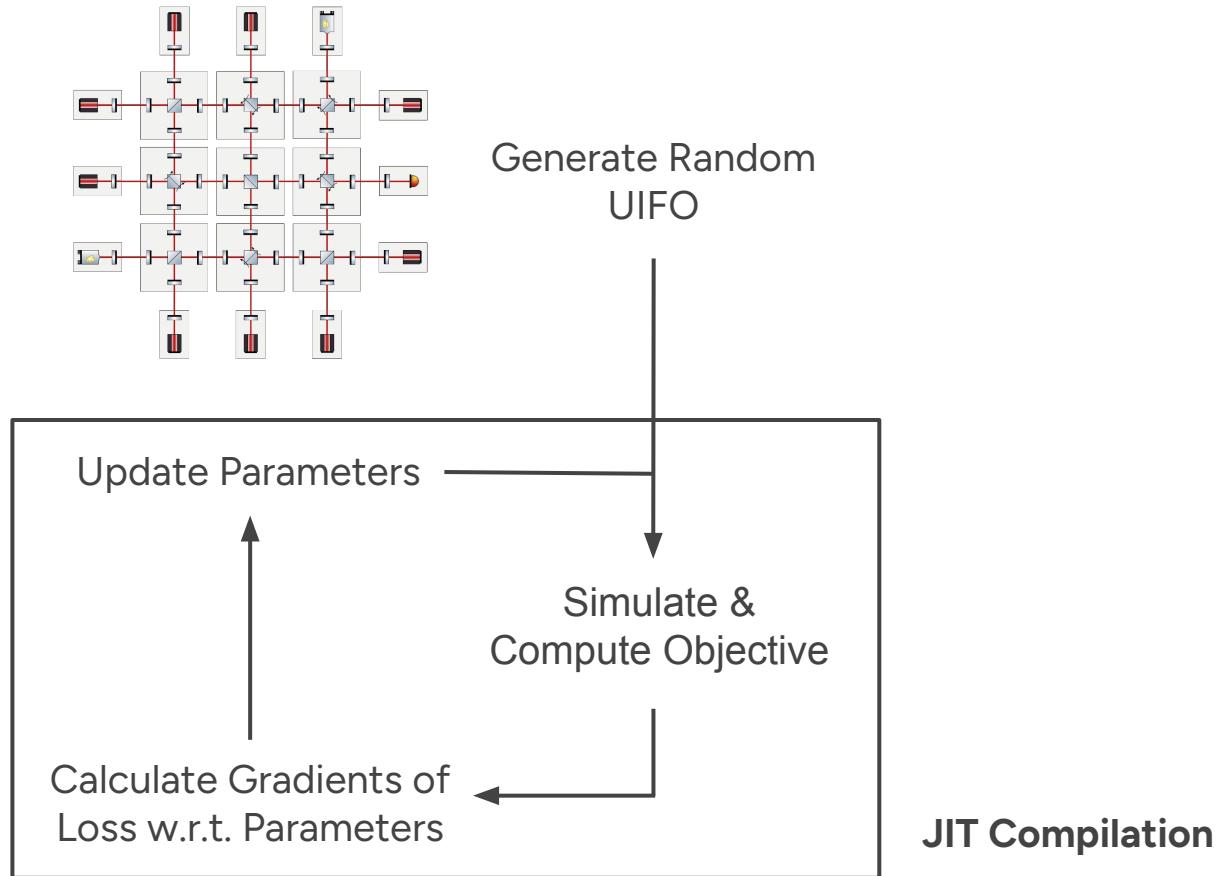
# Rewriting Finesse from Scratch in



# JIT Compiled Gradient Calculation

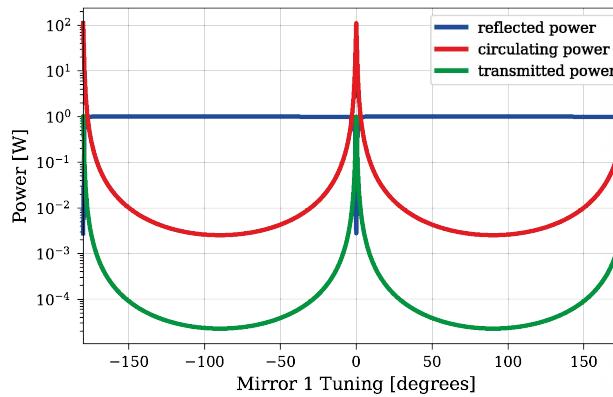
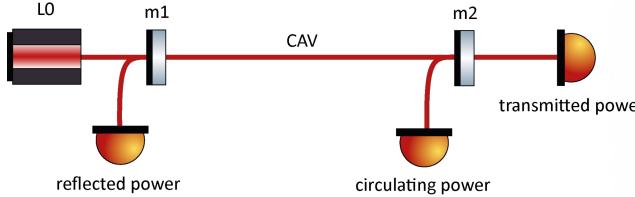


# JIT Compiled Gradient Calculation



# Differometer - Differentiable Interferometer Simulator

```
1 import differometer as df
2 import jax.numpy as jnp
3 from differometer.components import power_detector
4 import matplotlib.pyplot as plt
5
6 # define a simple cavity setup with three detectors
7 S = df.Setup()
8 S.add("laser", "l0", power=1)
9 S.add("mirror", "m0", reflectivity=0.99, loss=0)
10 S.add("mirror", "m1", reflectivity=0.991, loss=0)
11 S.space("l0", "m0", length=1)
12 S.space("m0", "m1", length=1)
13 S.add("detector", "refl", target="m0", port="left", direction="out")
14 S.add("detector", "circ", target="m1", port="left", direction="in")
15 S.add("detector", "trns", target="m1", port="right", direction="out")
16
17 # set the tuning range
18 tunings = jnp.linspace(-180, 180, 400)
19
20 # run the simulation with the tuning as the changing parameter
21 carrier, signal, noise, detector_ports, *_ = df.run(S, [("m0", "tuning")],
22   ↳ tunings)
23
24 # calculate the power
25 powers = power_detector(carrier)
26
27 # plot the power at the detector ports
28 plt.figure()
29 plt.plot(tunings, powers[detector_ports[0]], label="refl")
30 plt.plot(tunings, powers[detector_ports[1]], label="circ")
31 plt.plot(tunings, powers[detector_ports[2]], label="trns")
32 plt.yscale("log")
33 plt.xlabel("Tuning (degrees)")
34 plt.ylabel("Power (W)")
35 plt.grid()
36 plt.legend()
37 plt.tight_layout()
38 plt.savefig("output_cavity.png")
```



# Differometer Components

## Optics:

- Mirror
- Beamsplitter
- Directional Beamsplitter

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- Mirror
- Beamsplitter
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## Sources:

- Laser (only 0 Hz offset)
- Squeezer
- Signal Generator
  - Space Length
  - Laser Amplitude
  - Laser Frequency



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## Connectors:

- Space
- Nothing

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## Optics:

- Mirror
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- Directional Beamsplitter

## Detectors:

- Power Detector
- Demodulating Power Detector
- Quantum Noise Detector
- Balanced Homodyne Detector

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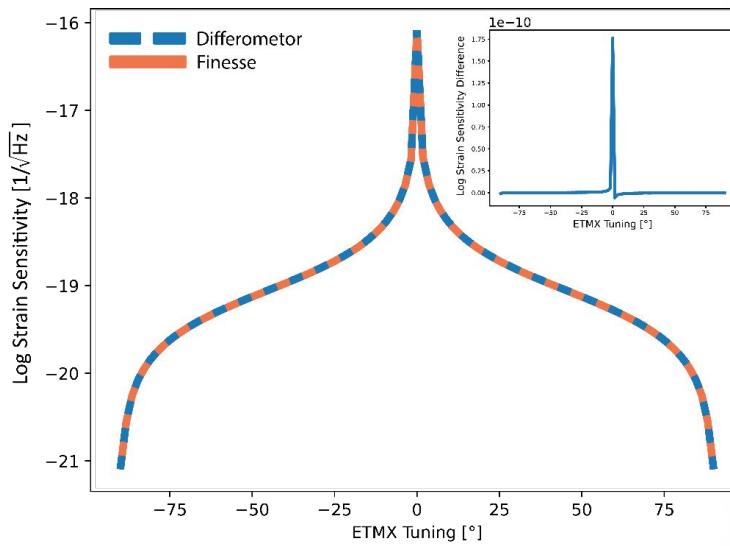
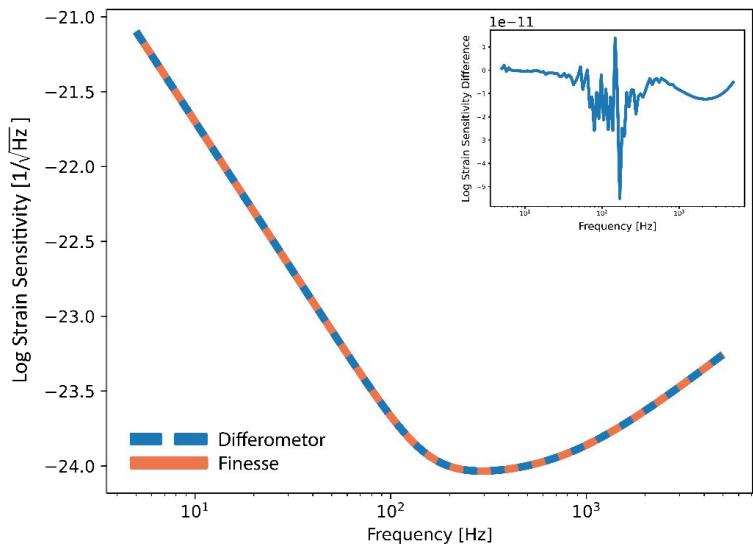
- Free Mass (only z node)

✗ Higher Order Modes

✗ Control Loops

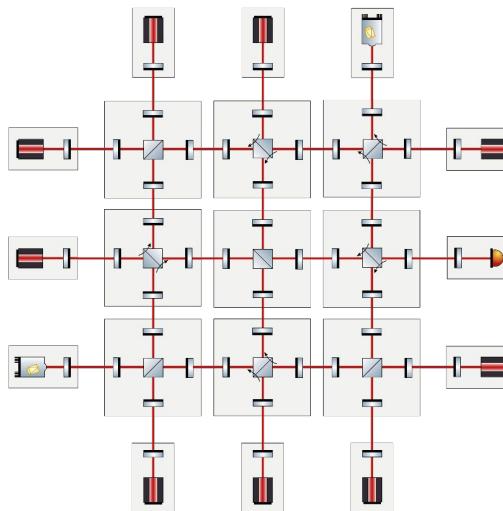
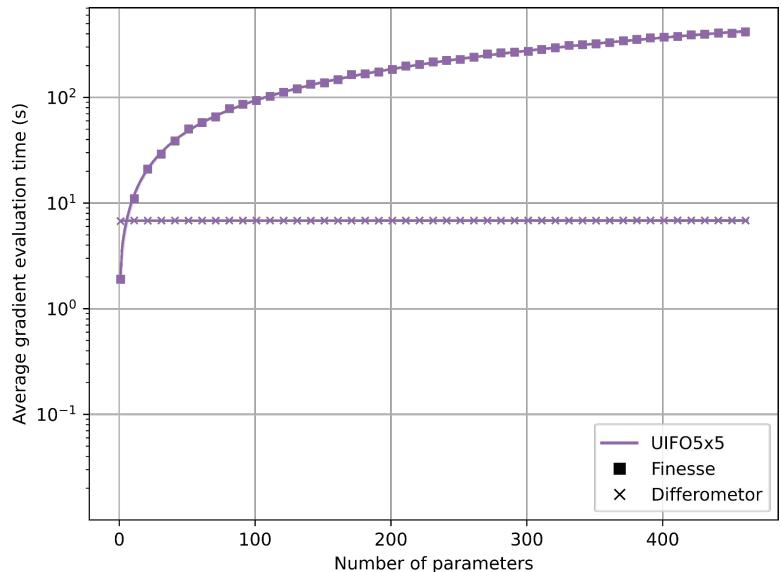
✗ ...

# Accuracy - Close Agreement with Finesse



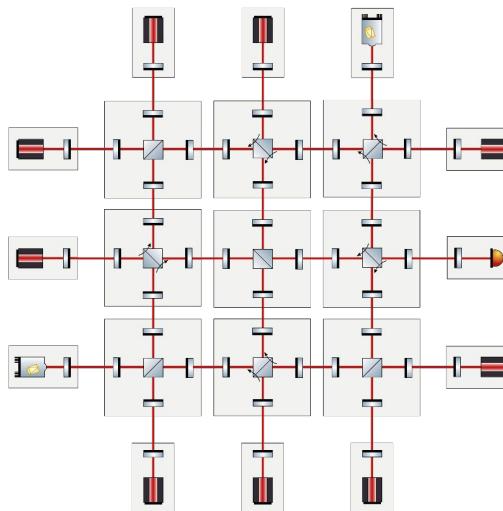
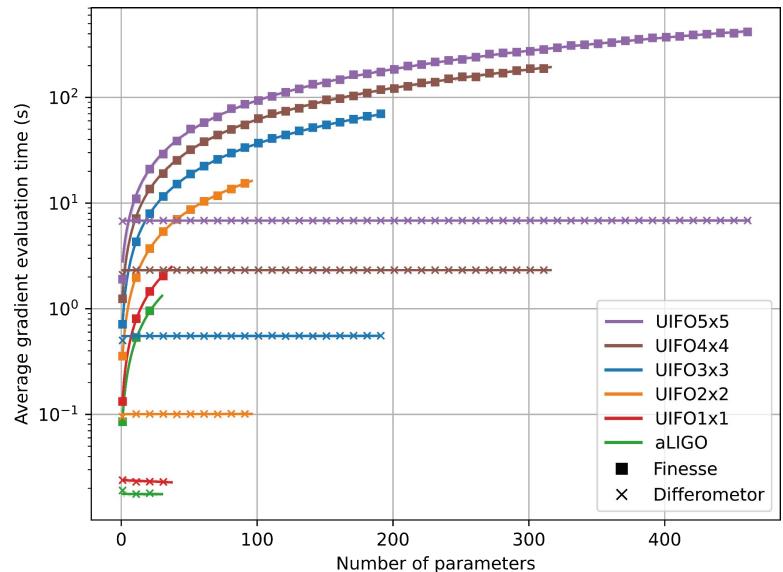
Simplified aLIGO

# Gradient Computation - Speed Gains through Autodiff



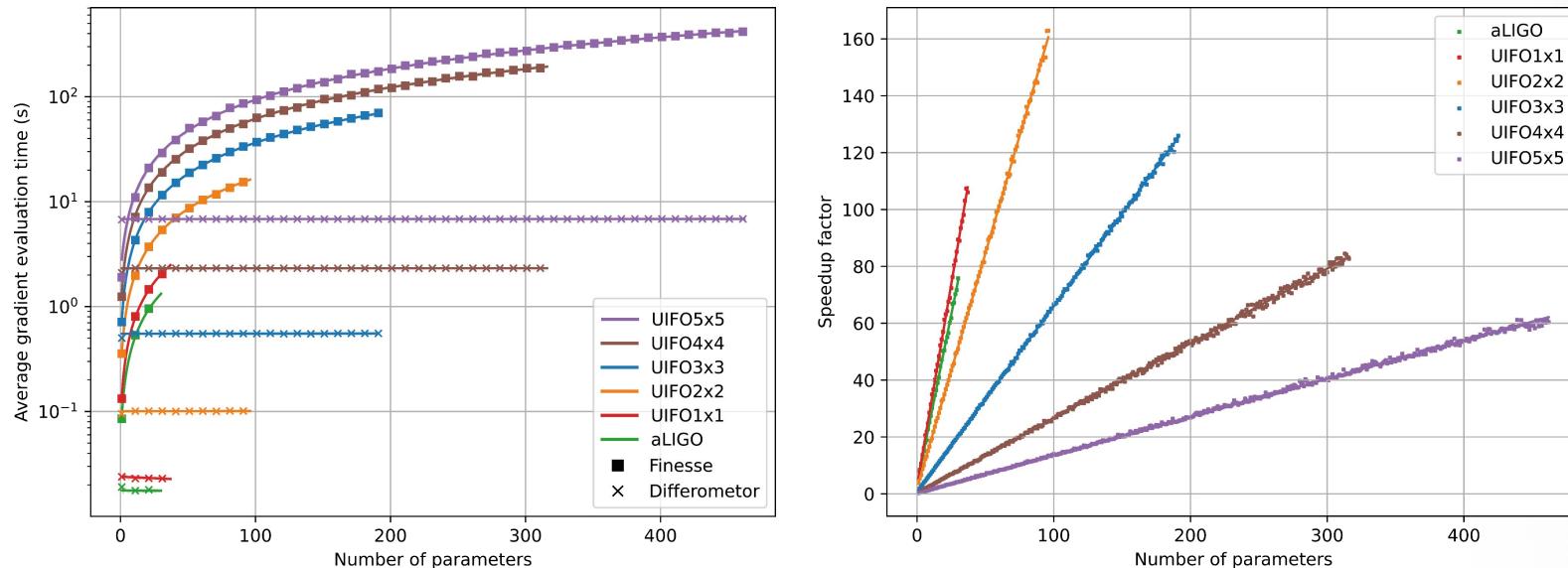
Nvidia Quadro RTX 6000 GPU vs. Intel Xeon Gold 6130 CPU

# Gradient Computation - Speed Gains through Autodiff



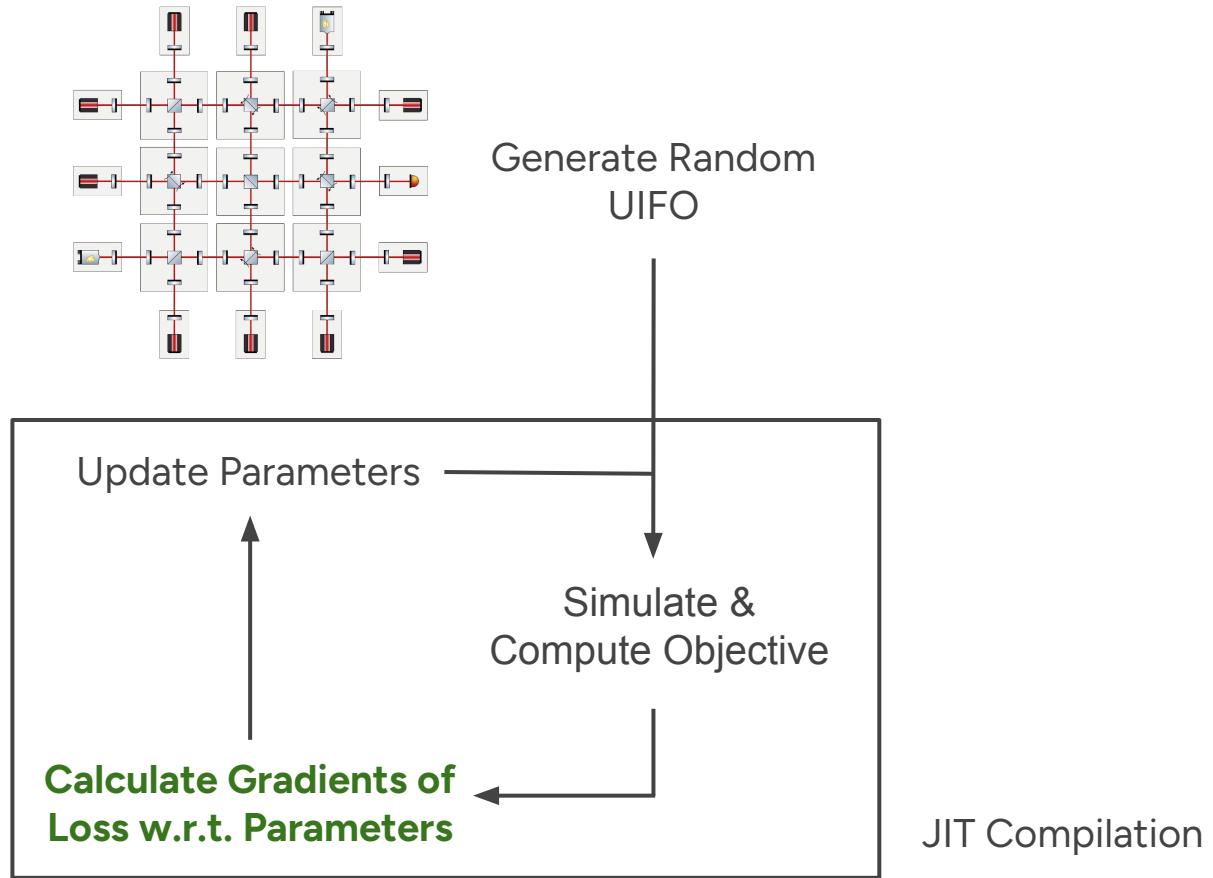
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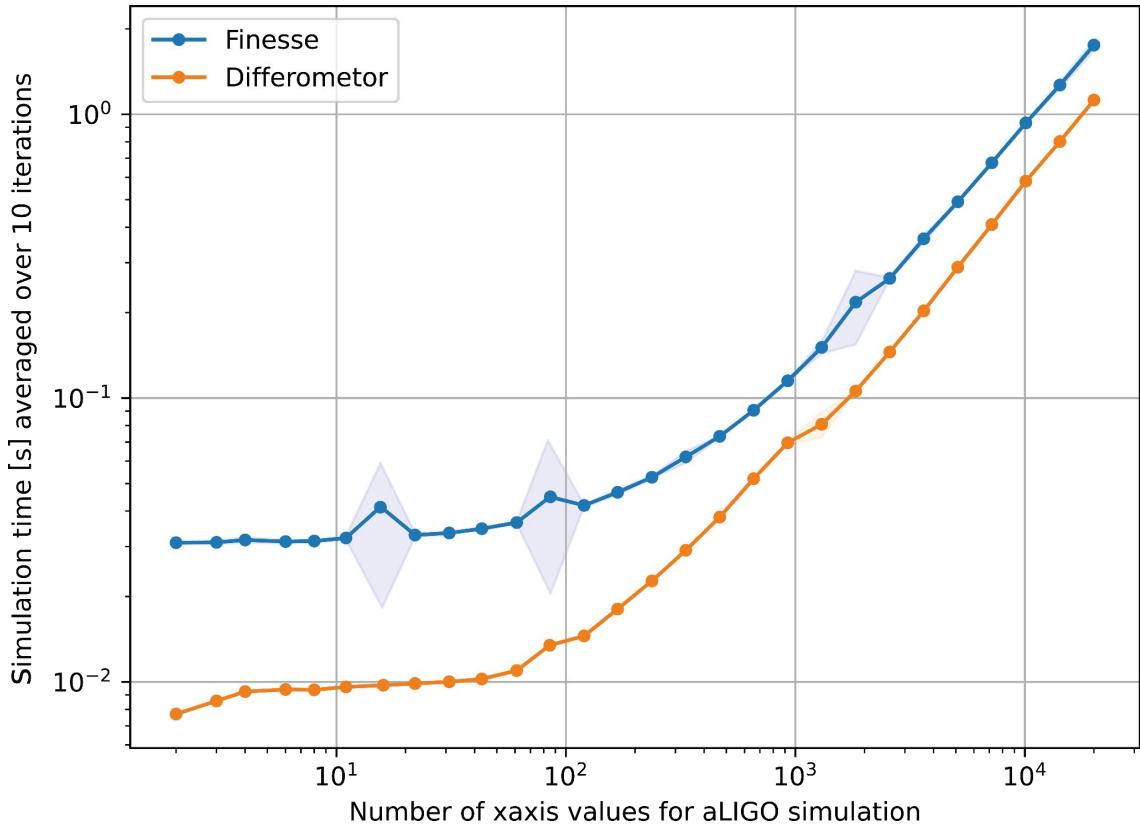


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# Gradient Computation - Speed Gains through Autodiff



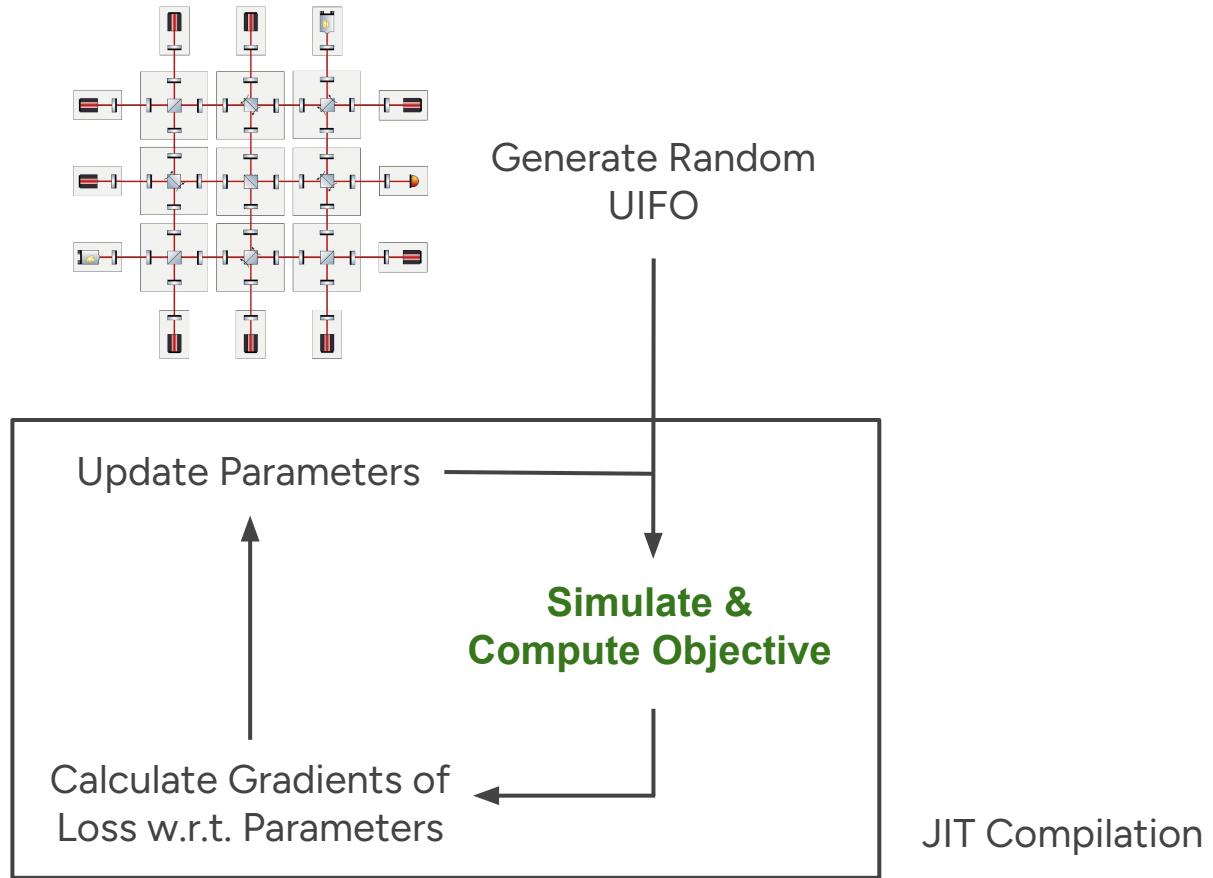
# Simulation - Speed Gains through GPU and JIT



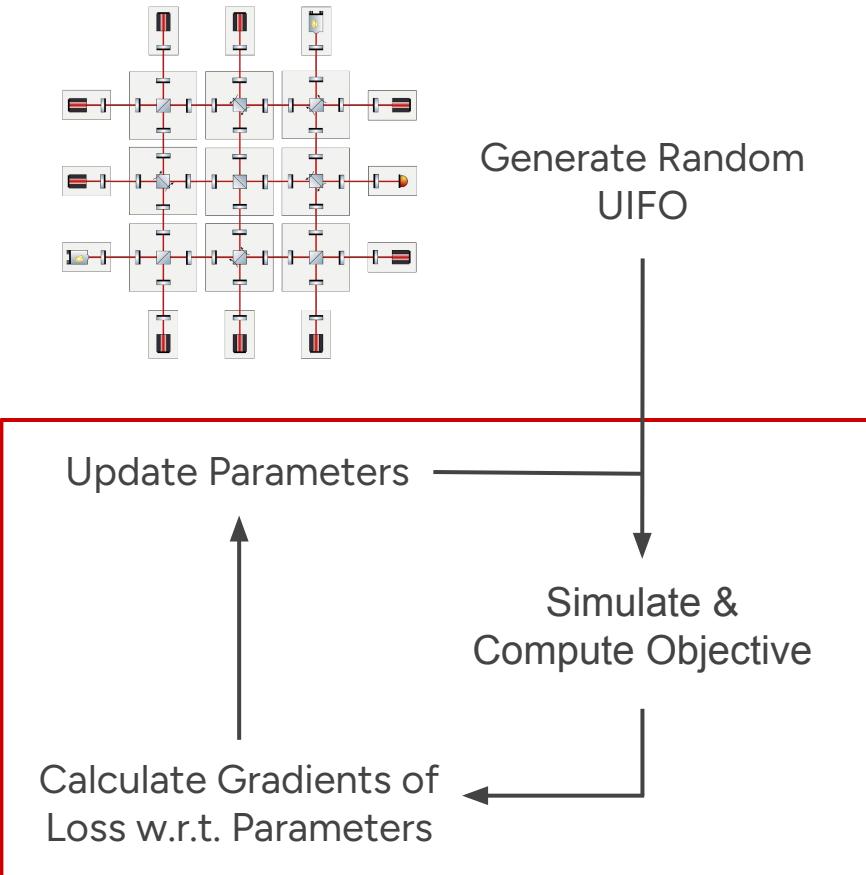
Simplified aLIGO

Nvidia Quadro RTX 6000 GPU  
vs. Intel Xeon Gold 6130 CPU

# Simulation - Speed Gains through GPU and JIT



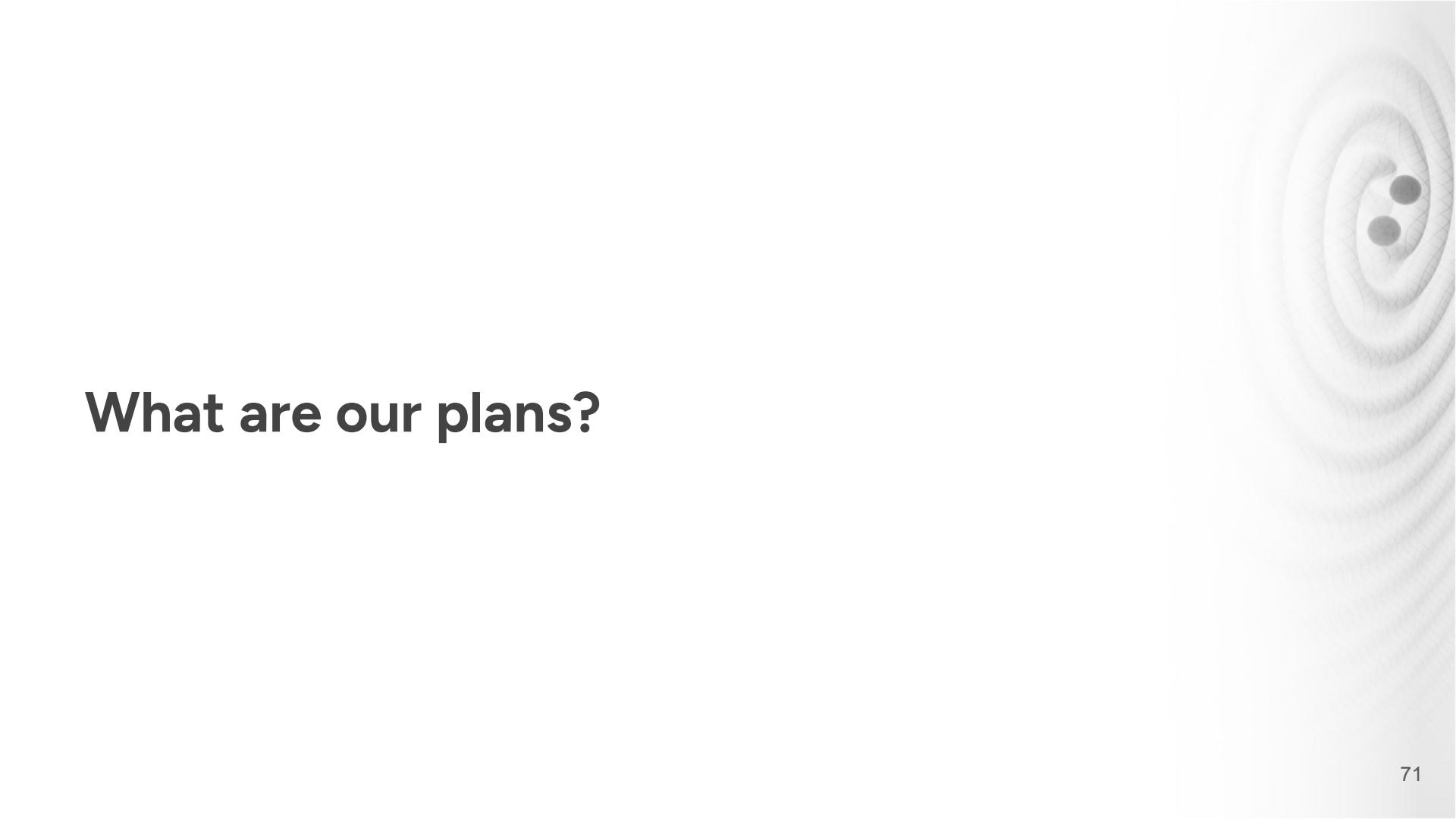
# Differomotor Compilation is Slow



## Simplified aLIGO:

- Differomotor: 40.4 s
- Finesse: 1.4 s

## JIT Compilation

The background features a minimalist abstract design. On the right side, there is a series of concentric, light gray circles that resemble ripples or waves. In the upper right quadrant, there are two small, dark gray circular dots positioned vertically, one above the other.

# What are our plans?

# Mario's ERC Grant



## Artificial Scientific Discovery of advanced Quantum Hardware with high-performance Simulators

Fact Sheet

### Project description

DE EN ES FR IT PL

#### Quantum experiment design with AI and high-performance simulators

The rapid advancements in generating, controlling, and measuring complex quantum systems have ushered in a new technological era based on quantum superposition and entanglement. However, designing these advanced experiments and hardware has grown too intricate for human researchers to handle alone. To unlock the vast potential of quantum mechanics, AI has been introduced into experiment design, but current methodologies remain limited. This creates a significant barrier to fully exploiting quantum physics' transformative possibilities. The ERC-funded ARTDISQ project aims to break these limitations by leveraging JAX, a cutting-edge computational framework, to build high-performance physical simulators. These simulators will power AI-driven breakthroughs in quantum-enhanced gravitational wave detection, advanced imaging techniques, and revolutionary quantum hardware design.

[Hide the project objective](#)

### Project Information

ArtDisQ

Grant agreement ID: 101165179

DOI

[10.3030/101165179](https://doi.org/10.3030/101165179)

EC signature date

6 December 2024

Start date

1 January 2025

End date

31 December 2029

Funded under

European Research Council (ERC)

Total cost

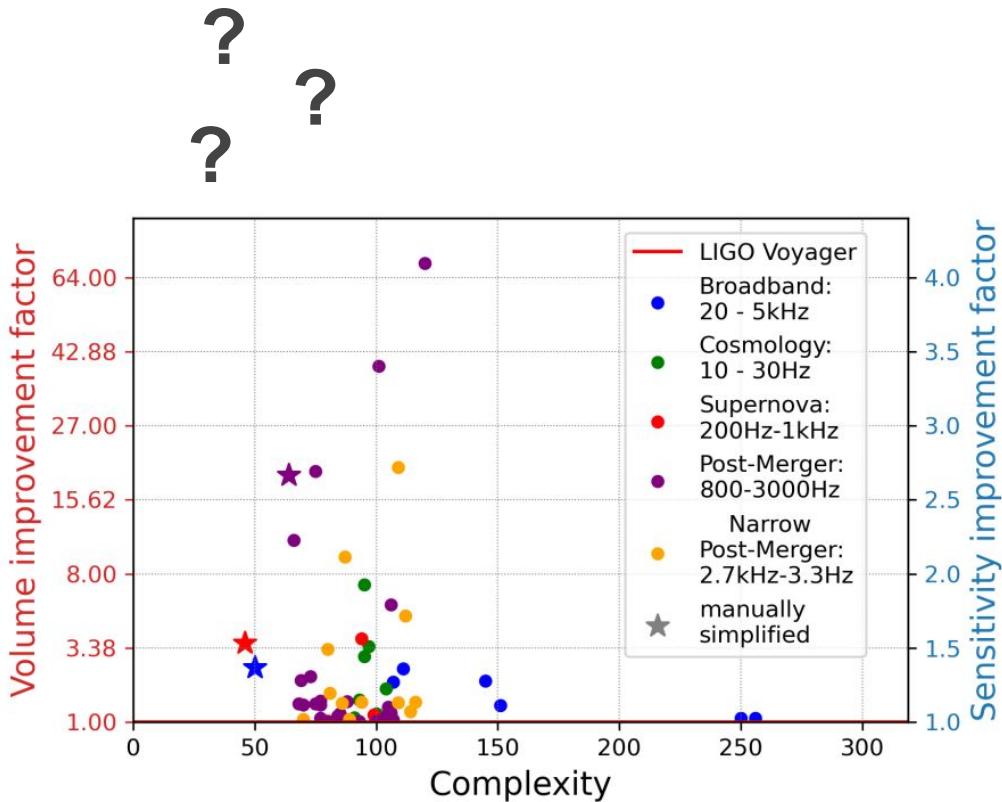
€ 1 499 221,25

EU contribution

€ 1 499 221,00



# Gravitational Wave Detector Zoo 2.0



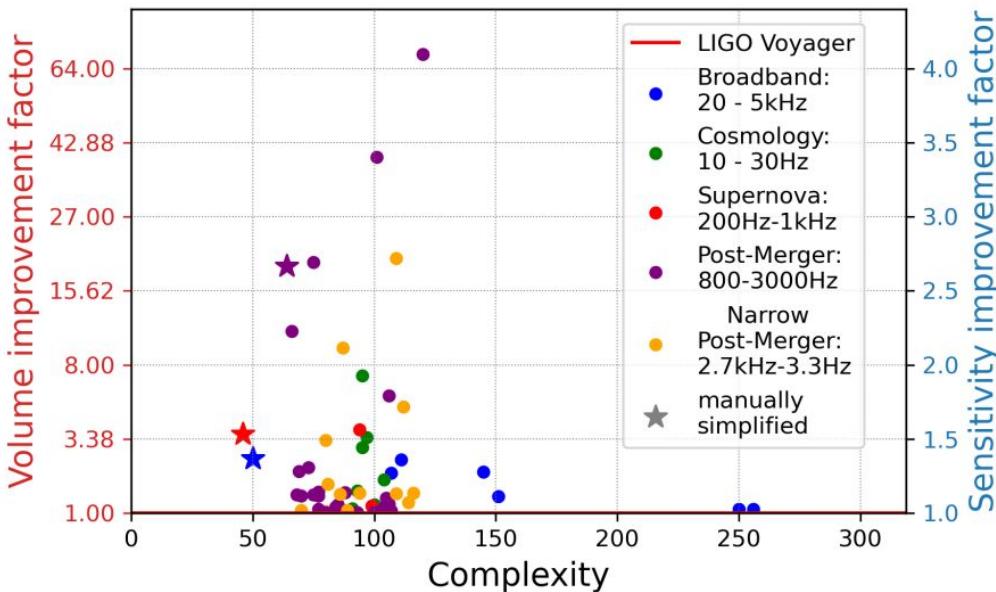
# Gravitational Wave Detector Zoo 2.0

?

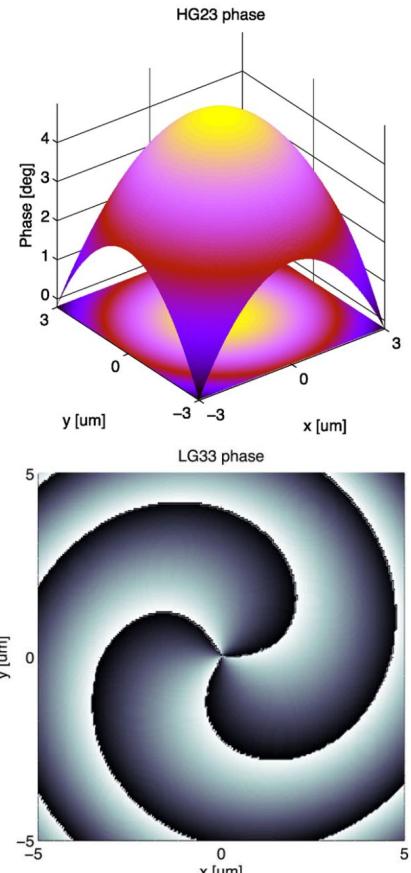
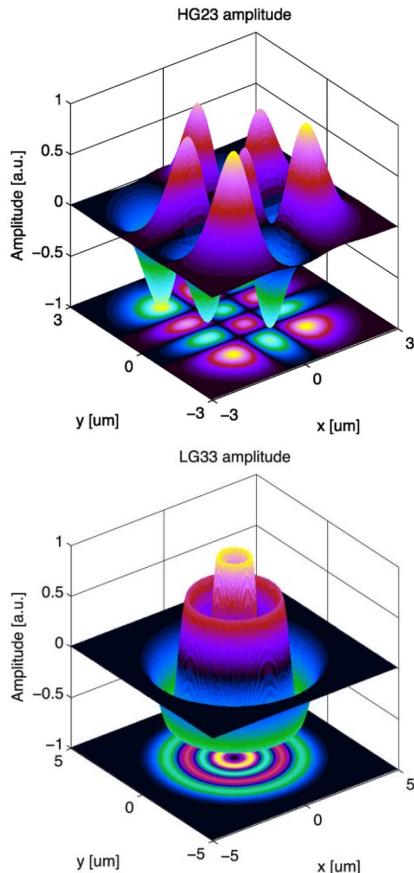
?

?

What are your thoughts on  
computational design?

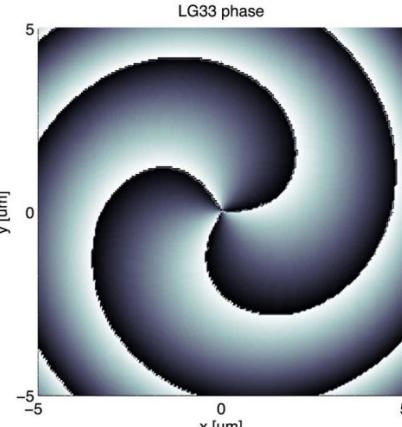
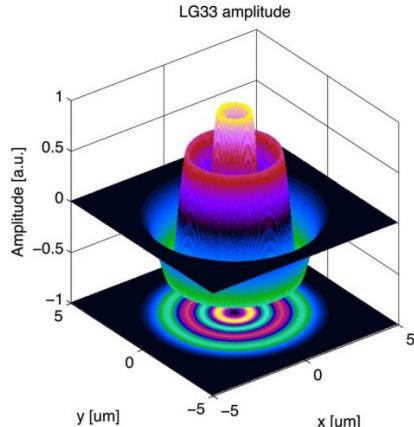
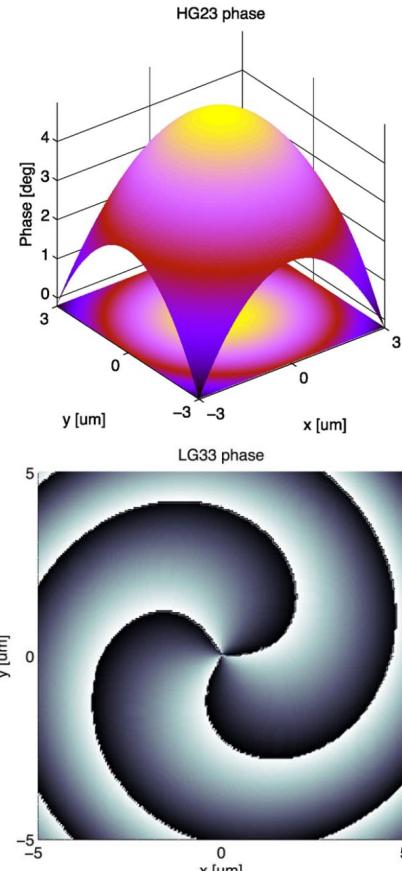
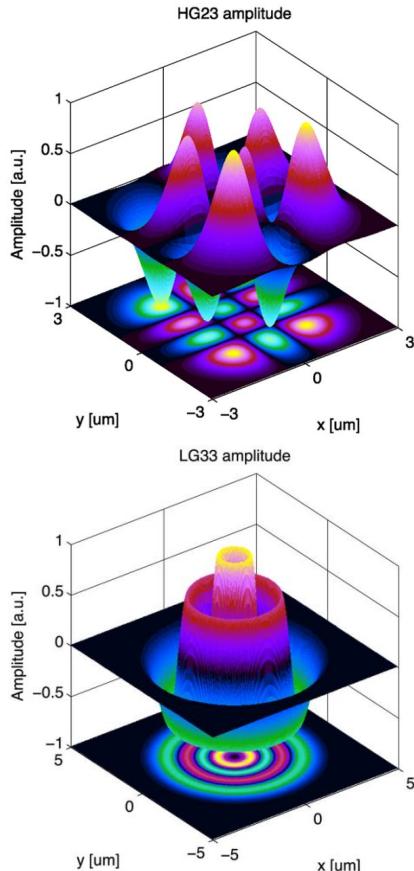


# New Features - e.g. Higher-Order Modes



Bond, C. et al. Living Rev Relativ 19, 3 (2016).

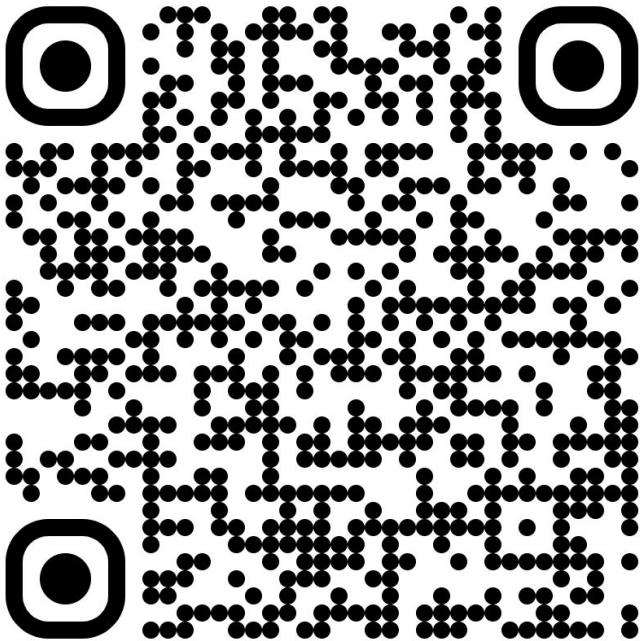
# New Features - e.g. Higher-Order Modes



Could Differometer help in some of your projects?

Which features should we implement next?

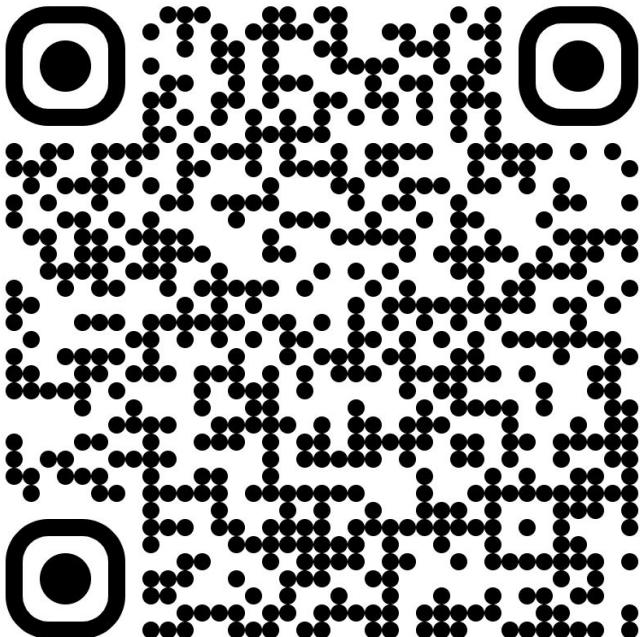
# `pip install differometer - try it out!`



<https://github.com/artificial-scientist-lab/Differometer>

- Differometer is a new differentiable interferometer simulator implemented in JAX
- It enables large speed gains in interferometer optimizations
- We use it for the computational design of new gravitational wave detectors

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- It enables large speed gains in interferometer optimizations
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**Thank you for listening!  
Any questions?**

# Backup

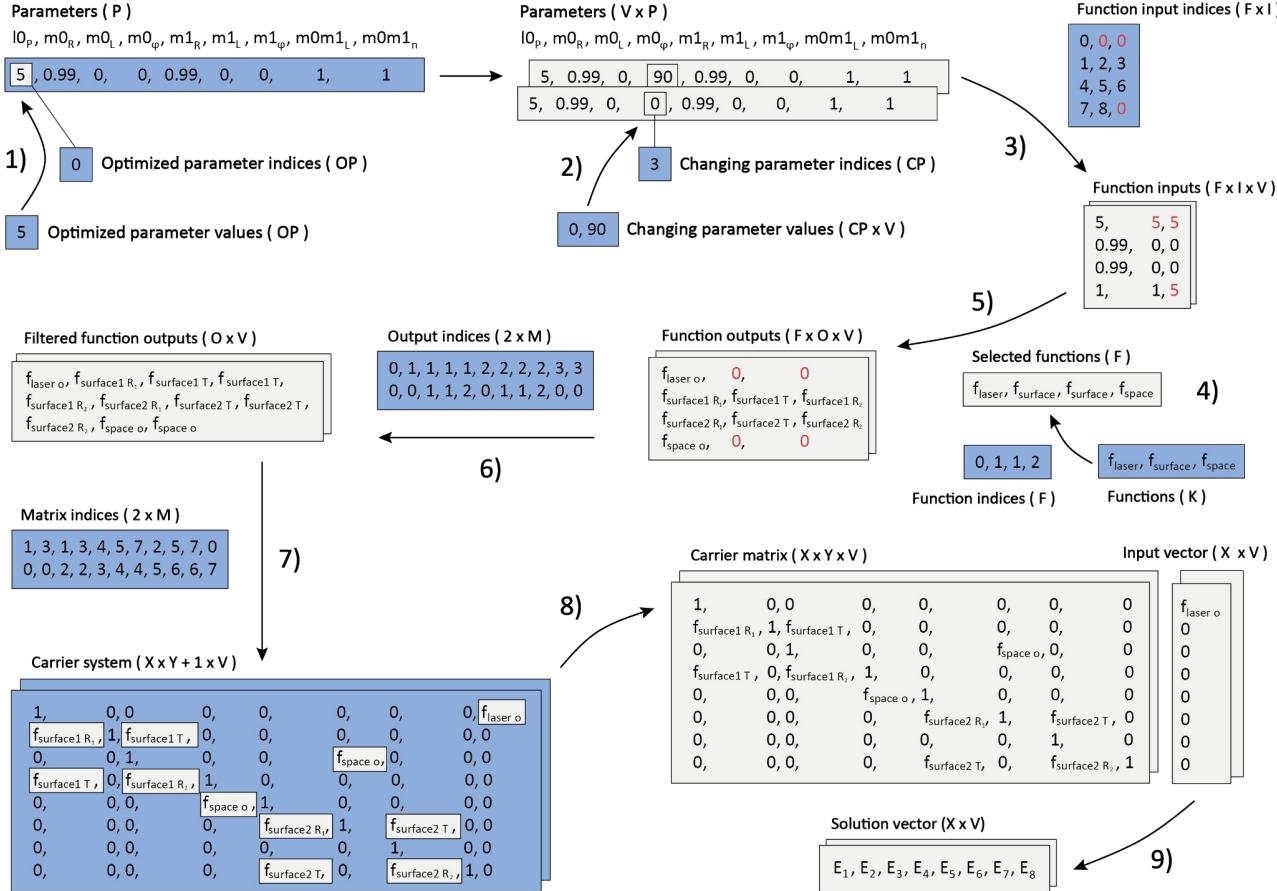
# Lines of Code Comparison

Finesse 3

Differometor

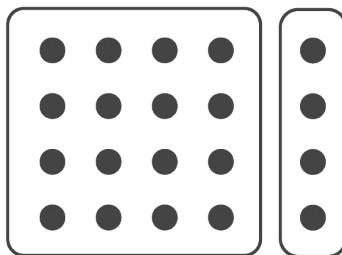
Files	269	8
Blank	14794	450
Comment	21736	646
Code	49152	2123

# Carrier System



# The Simulator Engine

System of  
Equations



# The Simulator Engine

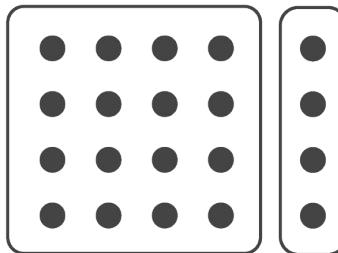
Parameters

$\Delta f$  ●  
 $L$  ●  
●  
●  
 $n$  ●  
●  
●  
●  
●  
●  
●  
●

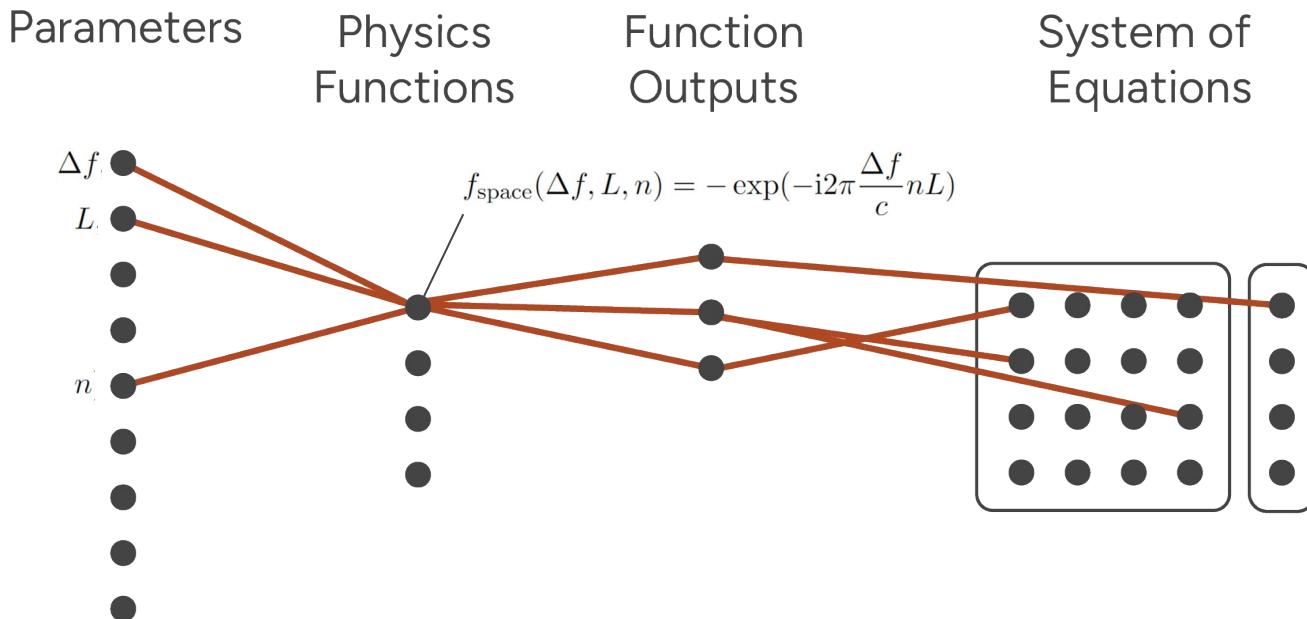
Physics  
Functions

$$f_{\text{space}}(\Delta f, L, n) = - \exp(-i2\pi \frac{\Delta f}{c} nL)$$

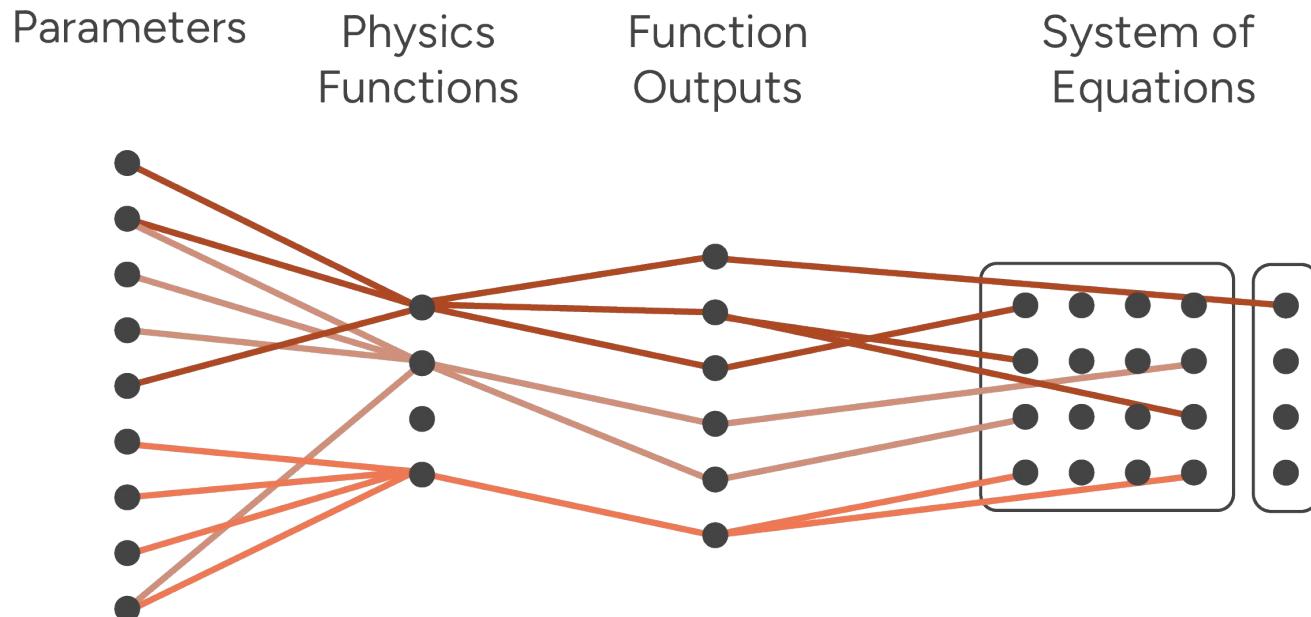
System of  
Equations



# The Simulator Engine



# The Simulator Engine



# Differometer - Subsystems

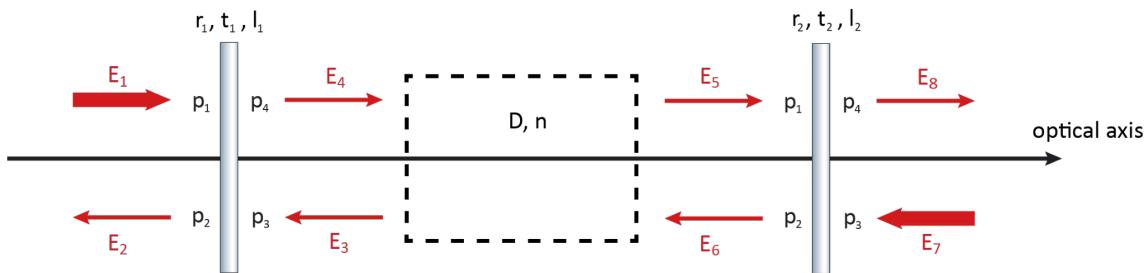
**Plane Wave  
Propagation**

Signal Sidebands

Quantum Noise

Optomechanics

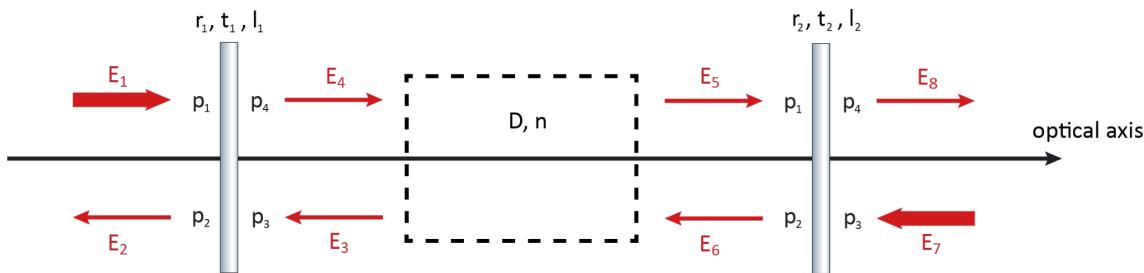
# Plane Wave Propagation



$$\begin{pmatrix} E_2 \\ E_4 \end{pmatrix} = \begin{pmatrix} it & r \\ r & it \end{pmatrix} \begin{pmatrix} E_3 \\ E_1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -r & 1 & -it & 0 \\ 0 & 0 & 1 & 0 \\ -it & 0 & -r & 1 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{pmatrix} = \begin{pmatrix} E_{1i} \\ 0 \\ E_{3i} \\ 0 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -r_1 & 1 & -it_1 & 0 \\ 0 & 0 & 1 & 0 \\ -it_1 & 0 & -r_1 & 1 \\ 0 & 0 & 0 & -e^{-iknD} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -e^{-iknD} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -r_2 & 1 & -it_2 & 0 \\ 0 & 0 & 1 & 0 \\ -it_2 & 0 & -r_2 & 1 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \\ E_7 \\ E_8 \end{pmatrix} = \begin{pmatrix} E_{1i} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E_{7i} \\ 0 \end{pmatrix}$$

# Differometer - Subsystems

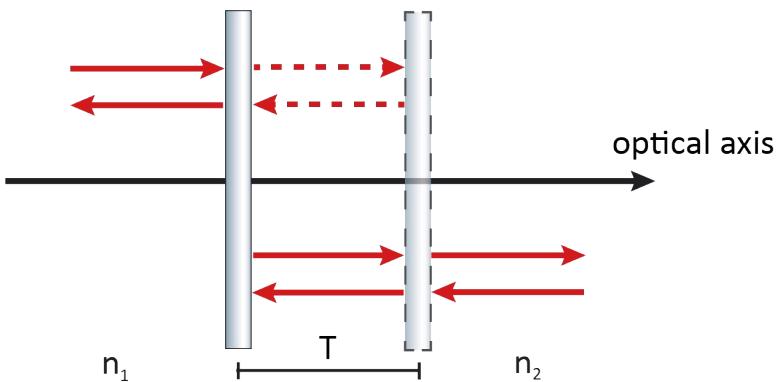
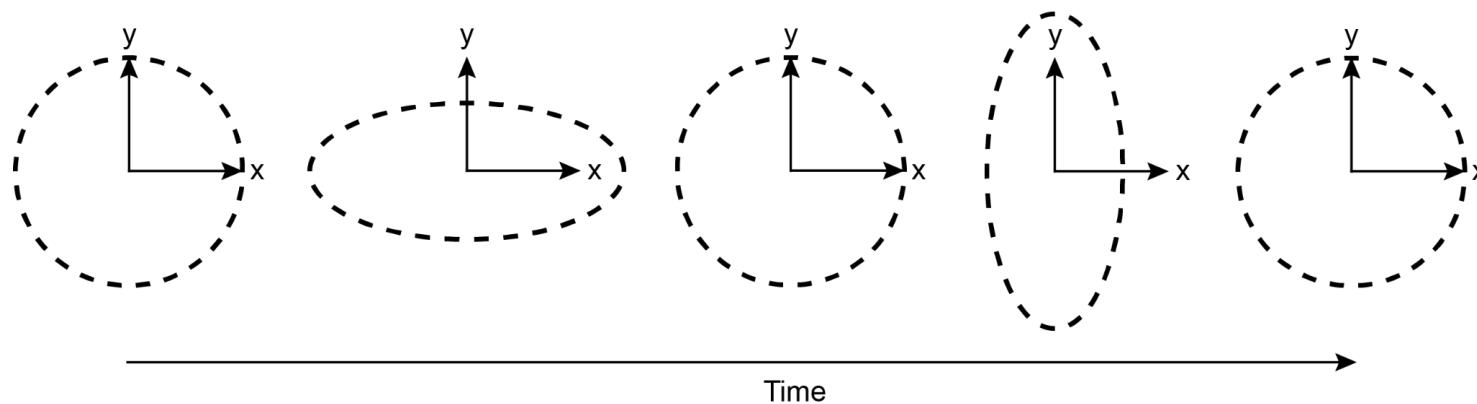
Plane Wave  
Propagation

**Signal Sidebands**

Quantum Noise

Optomechanics

# The Signal System



$$E = E_0 \exp(i(\omega_0 t + \varphi_0 + \phi(t)))$$

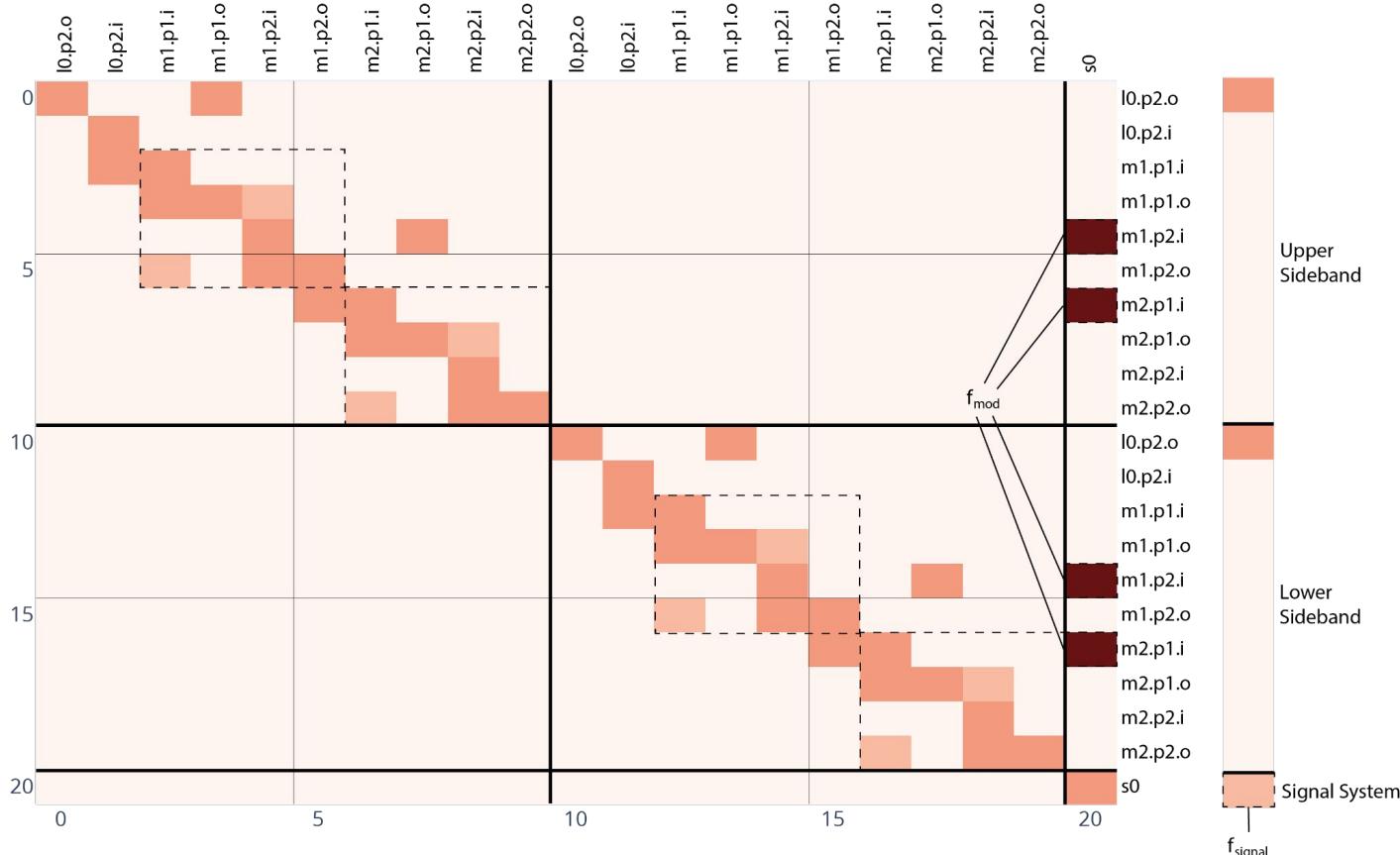
$$\phi(t) = m \cos (\Omega t + \varphi_s)$$

$$E = E_0 \left( 1 - \frac{m^2}{4} \right) \exp (i (w_0 t + \varphi_0))$$

$$+ E_0 \frac{m}{2} \exp \left( i \left( (w_0 - \Omega) t + \varphi_0 + \frac{\pi}{2} - \varphi_s \right) \right)$$

$$+ E_0 \frac{m}{2} \exp \left( i \left( (w_0 + \Omega) t + \varphi_0 + \frac{\pi}{2} + \varphi_s \right) \right)$$

# The Signal System



# Differometer - Subsystems

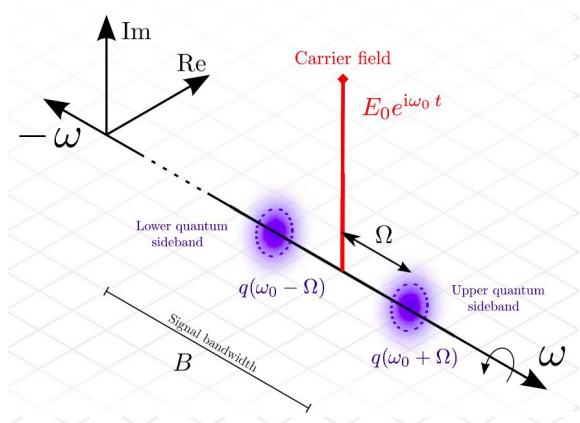
Plane Wave  
Propagation

Signal Sidebands

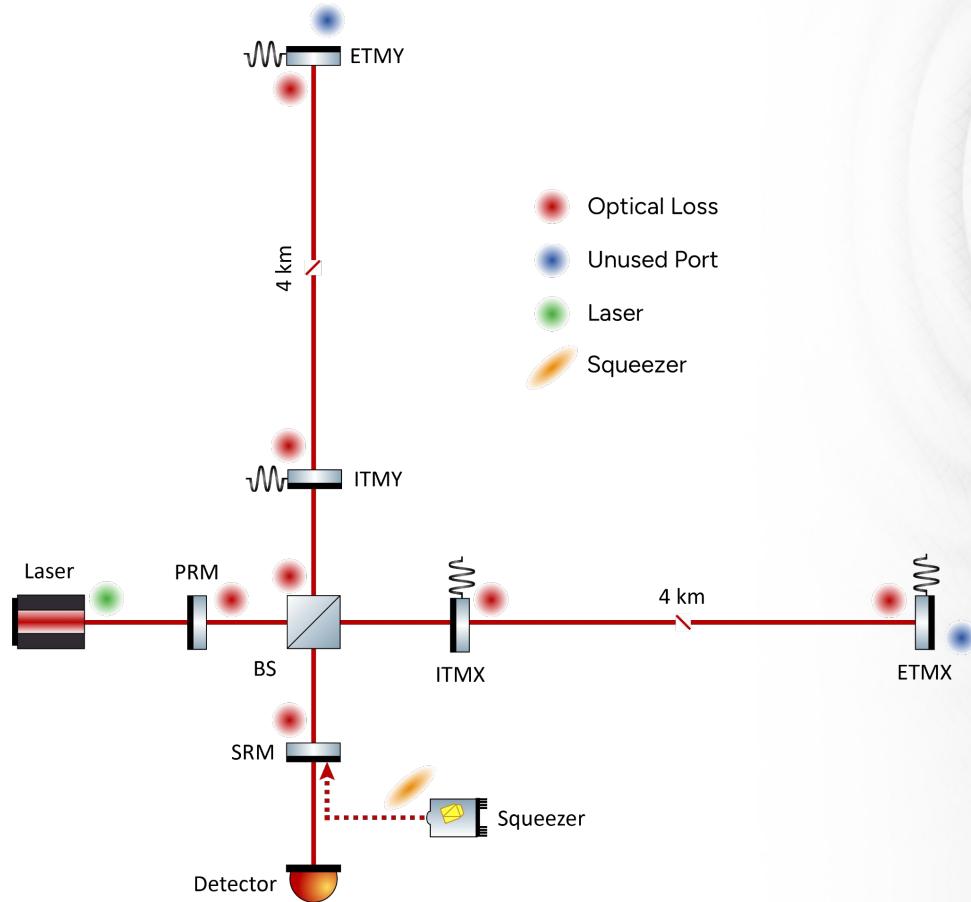
Quantum Noise

Optomechanics

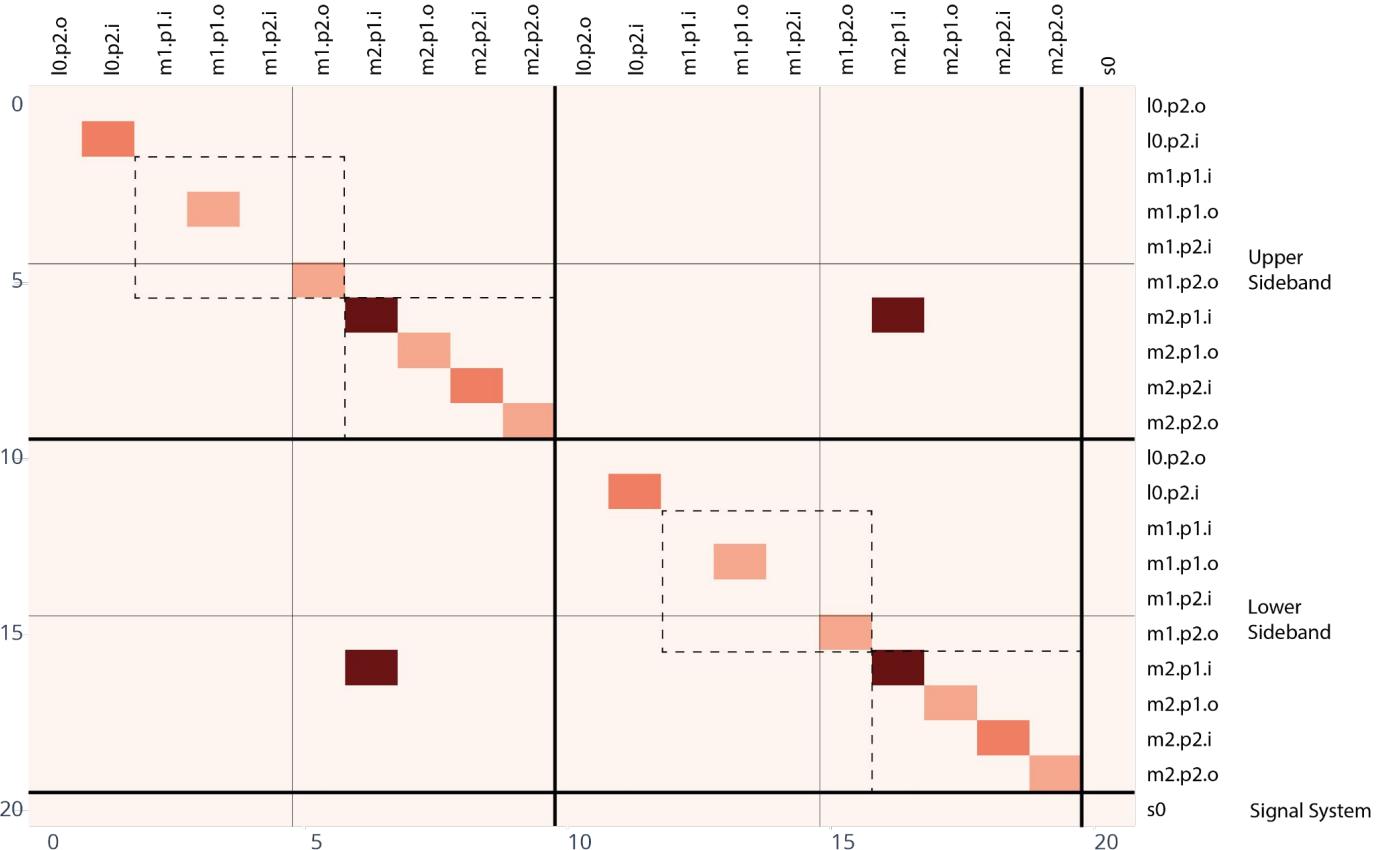
# Quantum Noise



$$\mathbb{V}_o = \mathbb{M}^{-1} \mathbb{V}_i \mathbb{M}^{-1\dagger}$$



# Quantum Noise



# Differometer - Subsystems

Plane Wave  
Propagation

Signal Sidebands

Quantum Noise

**Optomechanics**

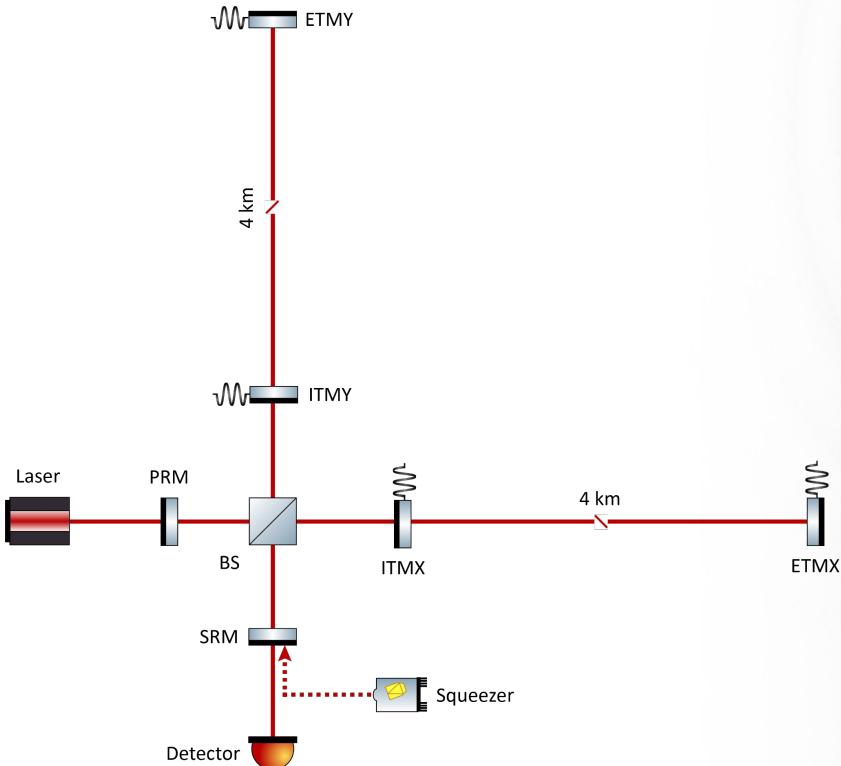
# Optomechanics

$$F(\omega) = \frac{P(\omega) \cos \alpha}{c}$$

$$H(\omega) = -\frac{1}{m\omega^2}$$

$$\delta z(\omega) = H(\omega) \sum_{n=0}^{N_f} F_n(\omega)$$

$$\varphi = -k\delta z(\omega)$$



# Optomechanics

